

Math. A.

50m - 1

Buchanan

PRACTICAL ESSAYS
ON
MILL WORK
AND OTHER
MACHINERY

ROBERTSON HENDERSON, ESQ.

OF THE SCOTCH MILLS

WITH NOTES AND OBSERVATIONS ON THE

CONSTRUCTION AND MANAGEMENT OF MILLWORK

BY THOMAS TAYLOR

OF THE SCOTCH MILLS

AND OF THE SCOTCH MILLS

Illustrated by G. P. P. and J. P. P.

VOL. I.

OF THE SCOTCH MILLS, THE SCOTCH MILLS, THE SCOTCH MILLS,
THE SCOTCH MILLS, THE SCOTCH MILLS, THE SCOTCH MILLS,
THE SCOTCH MILLS, THE SCOTCH MILLS, THE SCOTCH MILLS,
THE SCOTCH MILLS, THE SCOTCH MILLS, THE SCOTCH MILLS,

THE SCOTCH MILLS

LONDON

PRINTED FOR J. TAYLOR

AT THE SCOTCH MILLS, 10, BROAD STREET

1822

M. I. L. W. O. R. K.

MACHINERY.

ROBERTSON BUCHANAN, ENGINEER.

The Second Edition Corrected.

WITH NOTES AND EXPLANATIONS.

CONTAINING NEW RESEARCHES INTO THE MECHANICAL PRINCIPLES.

BY THOMAS TREDGOLD.

CIVIL ENGINEER.

MEMBER OF THE INSTITUTION OF CIVIL ENGINEERS.

Illustrated by 20 Plates and numerous Figures.

VOL. I.

ON THE THEORY OF WHEELS, THE KINEMATICS OF WHEEL-MOTION,
THE SHAFTS, GUNDRILLS, AND OTHERS, AND
MACHINES, WITH SEVERAL EXPLANATIONS.

LONDON:

PRINTED FOR A. TAYLOR.

AT THE ARCHITECTURAL LIBRARY, 25, BOND STREET.

1853.

PRACTICAL ESSAYS
ON
M I L L W O R K
AND OTHER
MACHINERY,

BY
ROBERTSON BUCHANAN, ENGINEER.

The Second Edition Corrected,
WITH NOTES AND ADDITIONAL ARTICLES,
CONTAINING NEW RESEARCHES ON VARIOUS MECHANICAL SUBJECTS,
BY **THOMAS TREDGOLD,**
CIVIL ENGINEER,
MEMBER OF THE INSTITUTION OF CIVIL ENGINEERS.

Illustrated by 20 Plates and numerous Figures.

VOL. I.

ON THE TEETH OF WHEELS, THE NUMBERS OF WHEEL-WORK,
THE SHAFTS, GUDGEONS, JOURNALS, &c. OF MILLS AND
MACHINES, WITH SEVERAL USEFUL TABLES.

London:

PRINTED FOR J. TAYLOR,
AT THE ARCHITECTURAL LIBRARY, 59, HIGH HOLBORN.

1823.

1

**Bayerische
Staatsbibliothek
München**

Staatsbibliothek
München

10-10-41

1897

OXFORD:

OXFORD :

PRINTED BY BARTLETT AND HINTON.

CONTENTS

OF

VOL. I.

ESSAY I.

*On the Teeth of Wheels, comprehending Principles
and their Application in Practice to Mill-work
and other Machinery.*

	PAGE
General definitions, art. 1—9. - -	1
CHAP. I.—Of the principles of the configuration of the teeth of wheels, art. 10—20.	5
General principles, art. 10, 11 - -	5
Properties of epicycloid, art. 12—20 -	11
CHAP. II.—Of the application of the principles of the configuration of the teeth of wheels, art. 21.—[D. 43.] -	28
SECT. I.—Of spur geers, art. 22.—[C. 40.] -	28
Of the wheel and trundle, art. 23 -	29
* To describe teeth for ditto by circular arcs - - - - -	37
Of the wheel and pinion, art. 26 -	39
* To describe teeth for ditto by circular arcs - - - - -	43

	PAGE
Remarks on the friction of teeth, art. 27	44
* Of the length of teeth - - -	50
On stave-formed teeth - - -	62
* On stave-formed teeth - - -	65
On the real radius of wheels, art. 33	69
Of the internal pinion, art. 38 - -	80
Of the rack and pinion, art. 40 - -	83
* Of the rack and pinion - - -	85
SECT. II.—Of bevel geer, art. 41.—[D. 43.] -	89
* New principle of describing the teeth of bevelled wheels - - -	99
CHAP. III.—Of involute teeth, art. 44 - -	107
* Involute teeth, properties of - -	110
Supplementary observations, art. 45 -	112
Remarks on the friction of wheel work, and the forms best suited for teeth, by Dr. Young. art. 47—53 -	117
Supplementary definitions, art. 54—72	127
* On power, force, momentum, and me- chanical power - - -	134
APPENDIX, No. 1.—Advertisement - -	141
A practical inquiry respecting the strength and durability of teeth of wheels, used in mill work, art. 73 - -	143
General observations on the wheel work of mills, art. 74 - - -	144
Horses' power, art. 79 - - -	155
Force of men, art. 83 - - -	158
* On the power of men and horses, and their velocity to produce a maximum of effect - - - -	160

CONTENTS.

v

	PAGE
Table of wheels that have been executed in mill work with explanation, art. 86 - - - - -	169
Observations on the table, art. 87 -	172
Tables of pitches, art. 89—102 -	175
* Principles of the strength of teeth with rules for the thickness and pitch of teeth of cast iron and of wood -	195
* Of the strength of staves -	204
Table of the radii of wheels, from 10 to 300 teeth by Mr. Donkin -	206
* Of arranging the numbers for wheel work of mills, &c. -	209
Practical observations on making pat- terns for cast iron wheels, art. 104	219
Of the materials for patterns, art. 108 -	227
APPENDIX, No. II.—On the use of charts, and some further explanation of the construction of tables of pitches of wheel work, art. 109 -	230

ESSAY II.

*On the Shafts, Gudgeons, and Journals of Mills,
the kinds of Stress to which they are subject,
their Strength, Stiffness, and Proportion.*

Preface - - - - -	245
Introduction - - - - -	251

	PAGE
CHAP. I. SECT. I.—General description of shafts and modes of fixing gudgeons, art. 129—136 - - - -	255
SECT. II.—Of the kinds of stress to which shafts are subject, art. 137 - - - -	264
Roberton on the stress on shafts, art. 138—148 - - - -	266
* Additions on the stress on shafts and gudgeons - - - -	285
CHAP. II. SECT. I.—Of the strength of gudgeons to resist lateral pressure, art. 150, 151 - - - -	293
* Investigation of a new rule for the strength of gudgeons - - - -	295
SECT. II.—Of gudgeons for water wheels, art. 152—158 - - - -	299
SECT. III.—Of cast-iron gudgeons for various pur- poses, art. 159—161 - - - -	310
SECT. IV.—Of malleable or wrought iron gud- geons, art. 162—165 - - - -	313
CHAP. III. SECT. I.—Of the strength of journals when the stress is partly twisting and partly lateral stress, art. 166 - - - -	320
SECT. II.—Of proportioning journals to the stress upon them, art. 167—171 - - - -	322
* Investigation of new rules for journals	330
CHAP. IV. SECT. I.—On the bodies of shafts, art. 173—179 - - - -	335
SECT. II.—Of lateral stress, art. 180—187 - - - -	348
SECT. III.—Of torsion or twisting, art. 188 - - - -	353

CONTENTS.

vii

	PAGE
Of hollow axles, art. 189 - - -	353
* Strain from torsion compared with that from lateral stress - -	358
SECT. IV.—Of the proportion of shafts, art. 192—	
195 - - - - -	361
Cast-iron shafts, art. 194 - - -	363
* Strength of cylindrical shafts of cast iron with a table - - -	368
* Ditto of hollow shafts - - -	370
Wooden shafts, art. 196 - - -	372
Shafts subject to torsion, art. 201 -	375
* Strength of cylindrical shafts to resist torsion with a table - - -	377
Table of shafts, art. 203 - - -	383
APPENDIX.—Cohesive strength of different mate- rials, 204—207 - - -	385
* Table of experiments on the direct cohesion of metals - - -	394
* Table of experiments on the direct cohesion of alloys - - -	398

CONTENTS.

vii

1758

Of hollow axes, art. 189 - - - 253

* Elastic form torsion compared with

that from lateral stress - - - 254

Part. IV.—Of the proportion of shafts, art. 192—

193 - - - 261

* Strength of cylindrical shafts of cast

iron with a table - - - 268

* Ditto of hollow shafts - - - 270

Wooden shafts, art. 196 - - - 272

* Shafts subject to torsion, art. 201 - - - 274

* Strength of cylindrical shafts to resist

torsion with a table - - - 277

Table of shafts, art. 203 - - - 282

Appendix.—Cohesive strength of different mate-

rials, 204—207 - - - 283

* Table of experiments on the direct

cohesion of metals - - - 294

* Table of experiments on the direct

cohesion of alloys - - - 308

GENERAL PREFACE

TO THE

FIRST EDITION.

WHILE my *Essay on the Teeth of Wheels* was in the Press, the idea occurred to me of publishing a series of Essays, all of a practical nature, on Mill Work, and other Machinery, each forming in itself a whole, at the same time that there would be a natural connexion, so that each Essay might again form a part of a general system. The Essay on the *Teeth of Wheels* I considered as the first of the series. The next in order that occurred to me was on the *Shafts of Mills*.

This is a subject of much practical importance; and it cost me much time and labour to collect and arrange facts respecting gudgeons and journals, with a view to deduce rules to guide the Millwright in a part of his business, which there is reason to suppose has hitherto in most cases been conducted at random. An Essay on this subject I accordingly sent to the Press in 1808, but when a considerable part of it was printed off, an unfortunate occurrence to the printer and publisher prevented it until now from being finished.

Meanwhile I had at intervals gone on, writing other Essays, some of which form the present volumes. The Essay *on the Shafts of Mills*, which with that on the Teeth of Wheels corrected, with additional tables, and a second appendix, form the first volume.

With a view to practical utility, I have endeavoured to adapt the style of these Essays to the comprehension of such operative mechanics as have not had the advantage of mathematical instruction; but at the same time I have given reference to authors for the demonstrations of such propositions as I found it necessary to introduce, in order to give such workmen some notion of the principles on which their work should be conducted.

For any repetitions, want of unity, and other imperfections, which will doubtless too readily appear in these Essays, I may offer the same apology which I did on a former occasion, that they were written at many different and distant intervals, occasioned by interruptions from professional and other engagements.

With a view to practical utility, I have endeavoured to adapt the style of these Essays to the comprehension of such operative mechanics as have not had the advantage of mathematical instruction; but at the same time I have given reference to authors for the demonstrations of such propositions as I found it necessary to introduce, in order to give such workmen some notion of the principles on which their work should be conducted. I have also given many repetitions, want of unity, and other imperfections which will doubtless be readily apparent in these Essays. I may offer the same apology which I did on a former occasion, that they were written at many different and distant intervals, and that I have been obliged to insert many corrections and additions from time to time, and that I have been obliged to insert many corrections and additions from time to time, and that I have been obliged to insert many corrections and additions from time to time.

PREFACE

TO THE

SECOND EDITION.

THE writings of the late Robertson Buchanan are well known among those who are interested in or follow mechanical pursuits. They contain an immense collection of practical information on a variety of subjects. They are particularly useful, because they give the arrangement and proportion of many parts of machines as they have been found useful or convenient in practice. If they sparkle not with the rays of superior genius, nor exhibit any remarkable degree of inventive talent, still they contain a full measure of useful truth; and he was the first to give such knowledge a popular form, or rather to rescue it from oblivion.

In this Edition a considerable quantity of new matter has been added; chiefly in the form of additional articles distinguished by the letters of the alphabet being prefixed to the numbers of the articles; these articles are illustrated by eight new cuts and one new plate. The most important of the additions are de-

scribed in the Table of Contents, and marked by a *. The other additions are in notes.

The brief account which follows, was selected from information obtained from the author's family through the kindness of Mr. Telford.

*A short Account of the Life and Writings of
Robertson Buchanan.*

ROBERTSON BUCHANAN was one of those individuals who rise to eminence through their own efforts; and it is desirable that some short notice of such men should be preserved, in order that others may be stimulated to make like exertions for the benefit of themselves and of their country.

He was born on the 14th of July, 1769, at Glasgow, and was connected by birth with some of its principal citizens. His father was nephew to Mr. Neil Buchanan, who, in 1768, represented Glasgow in parliament; and his mother was daughter to Mr. Arthur Robertson, who was chamberlain of that city for a long period. He unfortunately lost his mother at his birth, and his father when he was only about fifteen. His father had not been fortunate in business, and he was left unprovided for. He had already shown some talent for drawing and mechanics, which caused his maternal uncles to place him with a house carpenter of Glasgow; he afterwards worked some time with a millwright, and then left Glasgow for London.

It does not appear that he remained long in London, but returned to Glasgow, and commenced business as a mill-wright. In 1791, he gave up this business to go to Bute as manager of the Rothesay cotton mill, with a small share in the concern. He obtained, in 1796, a patent for an improved pump, which is not liable to choke; and in the same year he wrote some papers which were published in the Repertory of Arts and Manufactures,—one on the “Improvement of Cattle Mills,” and another on “Preventing Carding Machines from injuring the health of those employed to attend on them.”

He left Bute in 1801, with his health much impaired by the anxiety of a responsible situation in a losing business; and returned to London, with a view of deriving some benefit from the pump he had invented. In this he does not appear to have succeeded; but he got introduced to Count Rumford and Professor Pictet, which directed his attention to heating rooms; and in 1807, he published an “Essay on Warming Mills and other Buildings by steam.” He had previously been engaged in preparing the “Essay on the Teeth of Wheels,” which forms the first Essay of this volume; but it was not published till 1808. In 1810, he published his work on heating buildings in an improved form, with the title of “Practical and Descriptive Essays on the Economy of Fuel and Management of Heat.” In 1814 appeared the six Essays on Mill-work, which with that on the Teeth of Wheels, constitute these volumes. He published in 1816 a

xvi PREFACE TO THE SECOND EDITION.

“Practical Treatise on Propelling Vessels by Steam.” Besides these separate works, and some small pamphlets, he contributed the article “Cotton Spinning,” and the “Life of Sir Richard Arkwright,” to the Edinburgh Encyclopædia.

He died in the forty-seventh year of his age, at Creech St. Michael, Somersetshire, on the 22nd of July, 1816.

He was a man of amiable character, with a strong sense of religious and moral duty, and was greatly respected by all who knew him. His practical knowledge in mechanics was extensive; he was a close and accurate observer, and extremely assiduous in collecting every fact or experiment which came under his notice. On these facts he reasoned, in general, with great accuracy and judgment; and always with a view to improvement in practice. He was happy in the choice of popular subjects; and though there be an obvious want of method in handling them, yet it is more than compensated by the variety of useful and interesting particulars which are so abundantly scattered through his works. (Ed.)

ON

THE TEETH OF WHEELS.

ESSAY I.

P R E F A C E.

LED from situation, as well as curiosity, to attend very minutely to some parts of practical mechanics, one of the objects which early attracted the notice of the author of the following short Essay, was the figure of the Teeth of Wheels. He observed, that, in forming these teeth, workmen followed rules for which they could assign no satisfactory reason. Nor did he then find in books the information he wanted: the subject seemed to him to require a detail and simplification, which no English writer, with whom he was acquainted, had given it.— Afterwards, indeed, he found that some French mathematicians had treated it with much attention. But their works, though sufficiently

clear to those who have studied mathematics, are too abstract to be of general utility. In the following Essay, therefore, such an elucidation of the subject has been attempted, as might render it plain to the operative mechanic—an object, which will appear the more important, the more we consider the great variety of useful purposes to which wheel-work is applied.

De La Hire and Camus, are the two French writers, who have treated most extensively this branch of mechanics.—From the work of the latter, who has written more accurately, and more fully, the author has borrowed largely; nor has he scrupled to take from others, whatever he found to suit his purpose, and to make the fullest use of the communications of his friends.

Of the methods followed, it will be sufficient to remark, that the subject naturally suggested these two general divisions—First, the Principles of the Configuration of the Teeth, of Wheels:—Secondly, the application of these to practice.

The first chapter contains the Principles—The second, their Application, with certain

modifications--1st to *Spur Geer*, under which are comprehended the *Wheel* and *Trundle*; the *Wheel* and *Pinion*; the *internal Pinion*, and the *Rack* and *Pinion*.—And, 2dly, to *Bevel Geer*.

A third chapter is added, which contains a manner of forming *Spur Wheels*, upon principles somewhat different from those considered in the preceding chapter.

In the following pages, no pretensions are made, either to invention or profound investigation. The writer has studied perspicuity alone, and will have completely attained his object, if he has only been fortunate enough to give such a view of the various kinds of teeth, as will enable the artist to form some judgment of their respective merits, and to execute any of them with accuracy and ease. For this purpose it has been his aim to divest every part of the subject of obscurity, and to accommodate it to those who possess not the advantages of a mathematical education. But he is far from saying, that they will not find some difficulties, particularly in the first chapter; nor will they, perhaps, fully understand the truths it contains, till they see their relation to practice pointed

out in the second.—He found, that without becoming exceedingly prolix, there was no avoiding the use of some mathematical terms, but of these he has given definitions, either as the terms themselves occur, or at the conclusion of the Essay.*

* This Preface was written several years before the translation of Camus was published.

GENERAL DEFINITIONS.

I.

1. WHEN two toothed wheels act upon one another, the greater is called the *Wheel*, and the lesser the *Pinion*.

II.

2. Instead of the pinion, the trundle is sometimes used, such as is here represented. It is likewise known by the names of lantern and wallower.

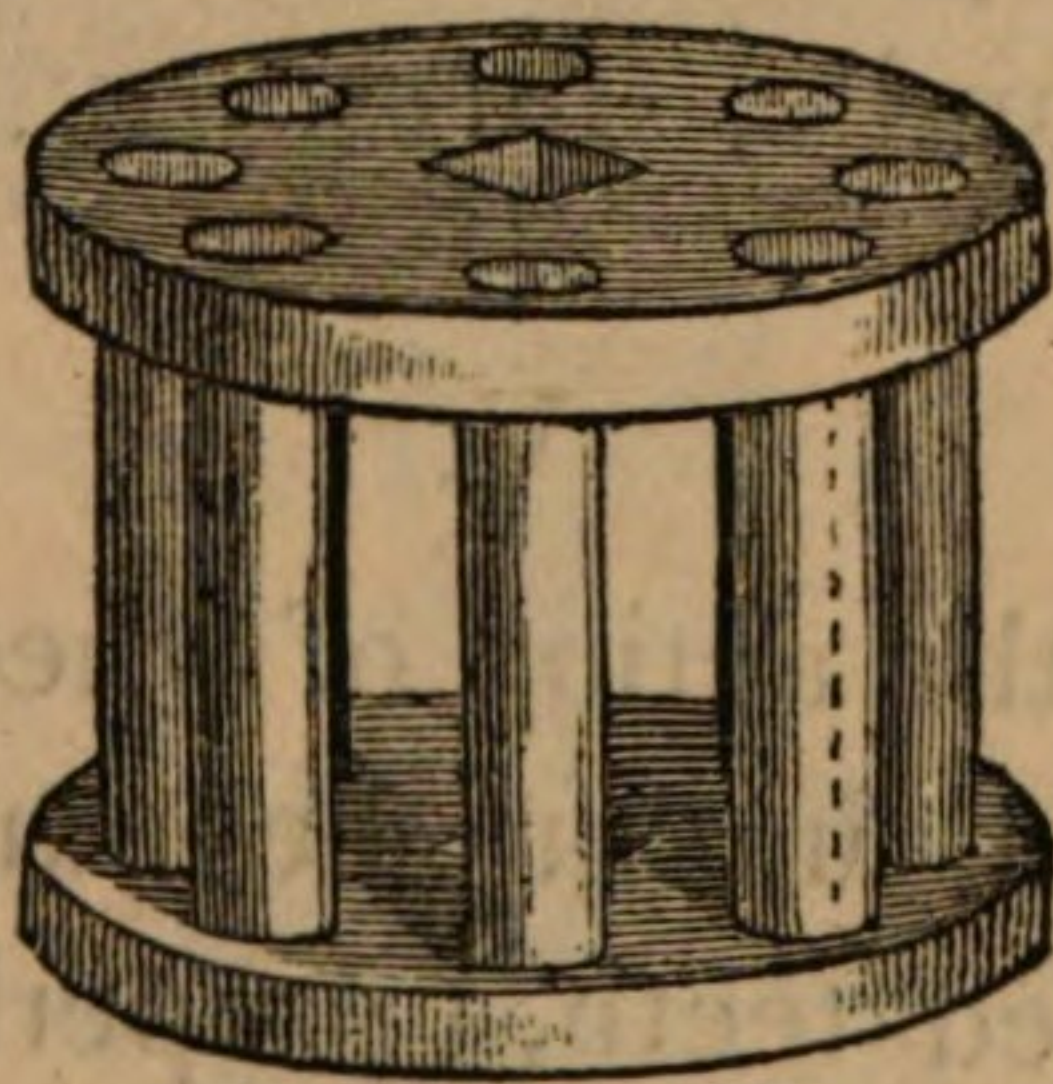


FIG. 1.

B

III.

3. As pinions and trundles are employed for the same purposes, when the action of two wheels is spoken of, in general, the trundle is comprehended under the name pinion.

IV.

4. The teeth of wheels and of pinions, are comprehended under the general term, *Teeth*. Where the teeth are of the same piece with the body of the wheel, they are called, properly, *teeth*; when they are each of a particular piece, they are called *cogs*. The teeth of pinions are called *leaves*, and those of a trundle, *staves*.

V.

5. When the action of wheels is spoken of in general, under the name *teeth*, are comprehended teeth, (properly so called,) *cogs*, *leaves*, and *staves*.

VI.

6. The straight line B F, which joins the centres B F, of a pinion and wheel, which act together, is called the *line of centres*.

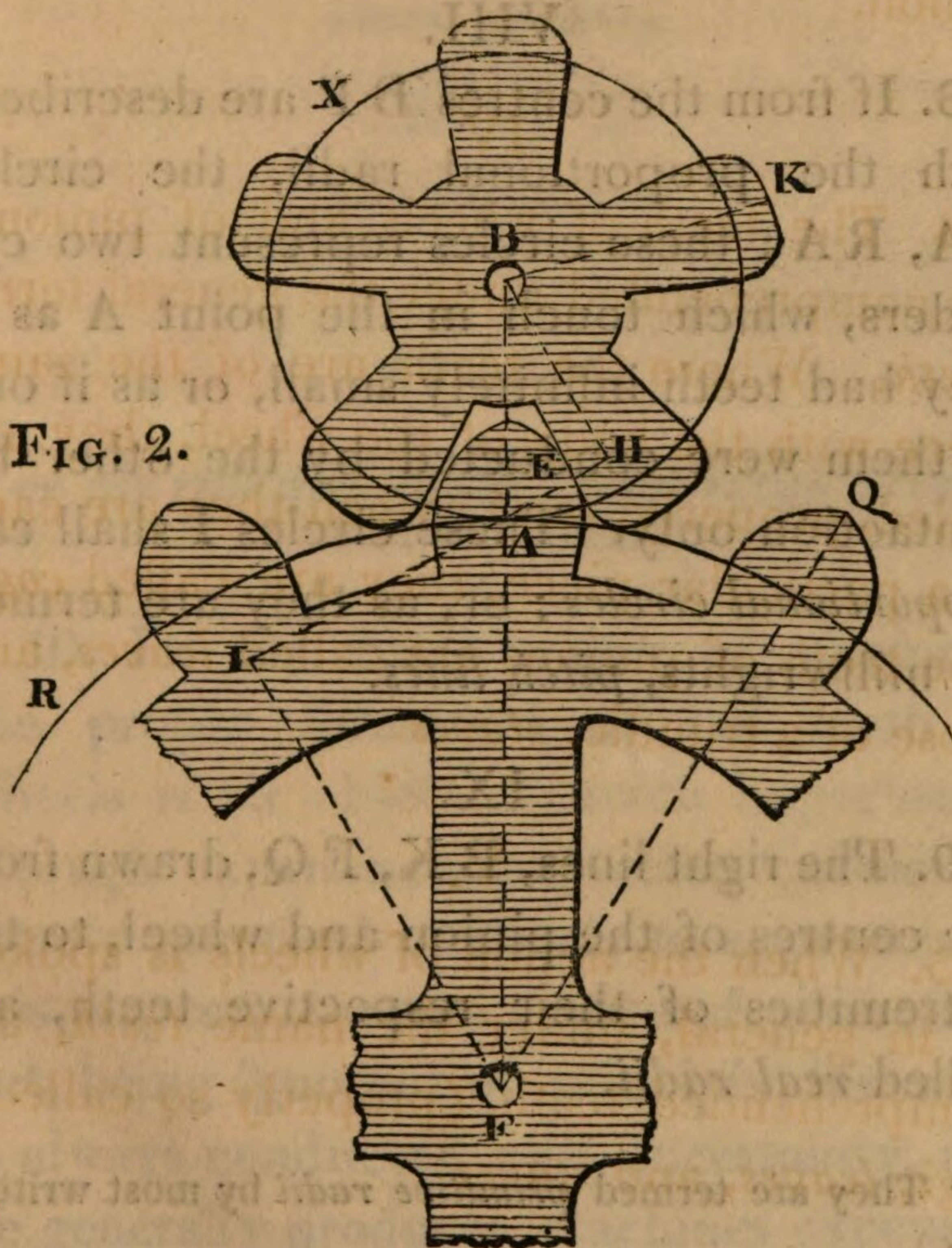


FIG. 2.

VII.

7. When the line of centres BF is divided into two parts, AB , AF , proportional to the number of the teeth in the wheel, and in the pinion, these two parts, AB , AF , are named *proportional radii*.*

VIII.

8. If from the centres B F are described, with the proportional radii, the circles XA , RA ; these circles represent two cylinders, which touch in the point A as if they had teeth infinitely small, or as if one of them were conducted by the other by contact only. These circles I shall call *proportional circles*; or, as they are termed by millwrights, *pitch lines*.

IX.

9. The right lines, BK , FQ , drawn from the centres of the pinion and wheel, to the extremities of their respective teeth, are called *real radii*.

* They are termed *primitive radii* by most writers.
(ED.)

ON
THE TEETH OF WHEELS.

ESSAY I.

CHAP. I.

**OF THE PRINCIPLES OF THE CONFIGURATION OF THE
TEETH OF WHEELS.**

10. IN the construction of machines, the proper formation of the teeth of wheels is an object of much importance. Though experience may often enable the merely practical mechanic to approach, in this respect, to some degree of perfection, yet, being ignorant of principle, his work is always conducted with uncertainty, and he generally produces machines expensive

in working, and defective in regularity, effect, and duration.

For when the acting parts of a machine are not truly formed, it may be so loaded as just to be in equilibrio with its work in the most favourable situation of its parts, but when it changes into a less favourable situation, the machine will stop, or at least, stagger, hobble, or work unequally.

The best figure, therefore, which can be given to the teeth, is that which shall cause them always to act equally and similarly, in situations equally favourable, and which shall consequently give the machine the property of being moved uniformly by a power constant and equal; or, in other words, ensure an uniformity of pressure and velocity.

Were the teeth of wheels infinitely small, their action would be regarded as that of cylinders, simply touching, having the property required. The finite and sensible

teeth given to wheels will, therefore, be of the most advantageous figure, when one wheel conducts another, as if they simply touched; or when their pitch lines have in every part of their revolution equal velocities.

That teeth have this property, when formed in a certain manner, will, I hope, be clear, from the following proposition and its connections.*

* The manner of forming teeth of wheels here referred to by our Author, would ensure an equable communication of power or motion in the imaginary case when the rubbing parts have no sensible friction; but in no other; except it be possible to contrive a practicable form for teeth having the property of rendering the friction uniform during the action of each pair of teeth: this has not yet been accomplished. Hence it appears that practical men have not without reason been doubtful of the advantages of the kind of teeth proposed by mathematical writers; for that the friction of teeth is not uniform, Dr. Young has proved in a letter, which forms a valuable part of this Essay, (see Art. 47—53.) And we have a practical proof of the same thing in the unequal wear of teeth, (see Art. 31.) The best means of avoiding the inequality produced by fric-

PROPOSITION.

11. When teeth are of such a form, that a perpendicular H E I* (Fig. 2. p. 3.) drawn to the tangent to the edge of the tooth in the point of contact E, cuts the line of centres at the termination A, of their proportional radii, *their pitch lines shall have in corresponding places, equal velocities*, whether the wheel drives the pinion, or the pinion the wheel; that is to say, that they will move each other as if they merely touched.†

tion seems to be, to make the teeth “as small and as numerous as is consistent with strength and durability,” (Art. 46.) These limits, with respect to strength and durability, I will endeavour to establish in the additions to Art. 88, and those following it. And since, in adopting the principle of short teeth, the curved surface of each tooth will become so small that a circular arc may be employed instead of the proper curve, I shall, in the additions, point out the mode of describing circular arcs to answer this purpose.—(ED.)

* For the manner of drawing this perpendicular, see Art. 17.

† This being a fundamental proposition, it is of importance that it should be well understood; I shall

I shall now proceed to show, that the epicycloid gives the property to the teeth of wheels required in the preceding proposition, and shall begin with some definitions respecting that curve.

Before we proceed with our Author, it will be an advantage to examine this proposition more particularly.

[A. 11.] Let AH (Fig. 2.) be the direction of the force of the wheel to turn the pinion, and BH a line perpendicular to AH , drawn to the centre of motion B . The effect of the

therefore, in this note, attempt a popular illustration of it.

It is demonstrable that the line HB (Fig. 2.) has the same proportion to the line IF which AB has to AF . Now let us suppose HB and IF to be levers, and HI a string, the one lever pressing from the other, would act upon it with just the same force that the pinion and wheel do at the point A , where the pitch lines touch; or, in other words, as if the circle X acted on the circle R , by means of a string, as pulleys do on each other by a band.

force to turn the pinion will be directly as the distance of its direction from the centre of motion, or as HB . Also, the angular velocity generated will be inversely as the distance of the direction from the same centre, or as $\frac{1}{HB}$. Consequently, the quantity of motion communicated to the pinion is as $\frac{HB}{HB}$; that is, in an invariable ratio; but by the same reasoning it may be proved that the force of the wheel at A is invariable; and therefore, the pinion will be moved in the same manner as if it were moved by contact at A , when HA is perpendicular to the common tangent of the surfaces in contact at E .

The same may be proved when the pinion drives the wheel. But this, as well as the more detailed investigations of Camus (on the Teeth of Wheels, Art. 521.) and his followers, neglects the effect of friction. Let the effect of the friction of the surfaces be represented by x , when the pressure and velocity of these surfaces are each equal unity, or 1; then the ratio will

be $\frac{HB}{HB(1-x)}$, or as $\frac{1}{1-x}$; which is invariable only when the friction is invariable. But when the teeth are very short, and formed so that the motion would be uniform were the friction uniform, it is perhaps the best practical method of forming teeth. (ED.)

DEFINITIONS.

I.

12. If upon the same immoveable plane are placed two circles, CNP, CALMK, (Fig. 3 and 4.) which touch each other in the point C, and the former, with a supposed style or tracer in its circumference at the point C, is made to revolve round the circumference of the latter, the style C, during the revolution, will describe upon the plane CALMK the curve CEGDK, which is called an *epicycloid*.

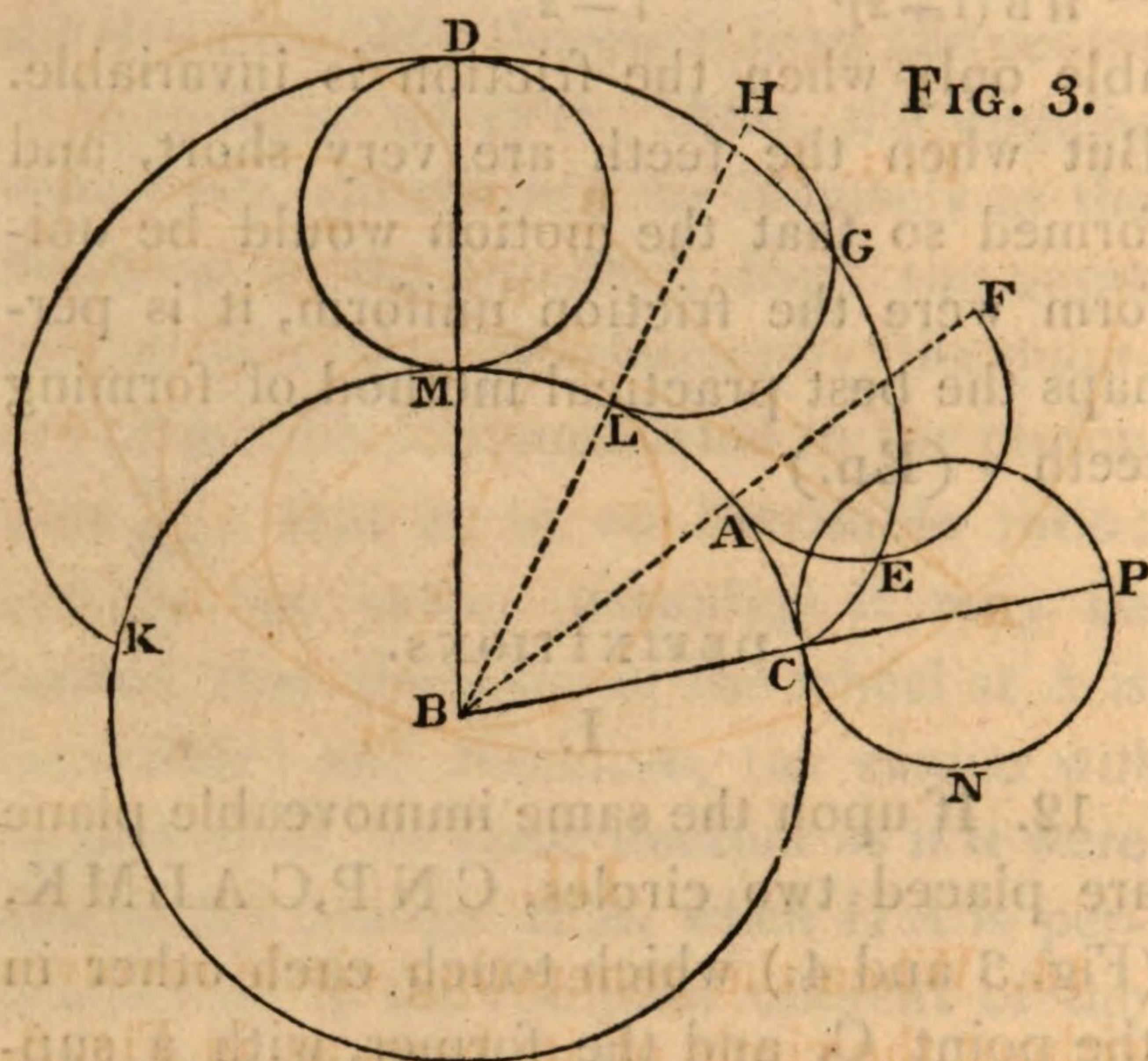
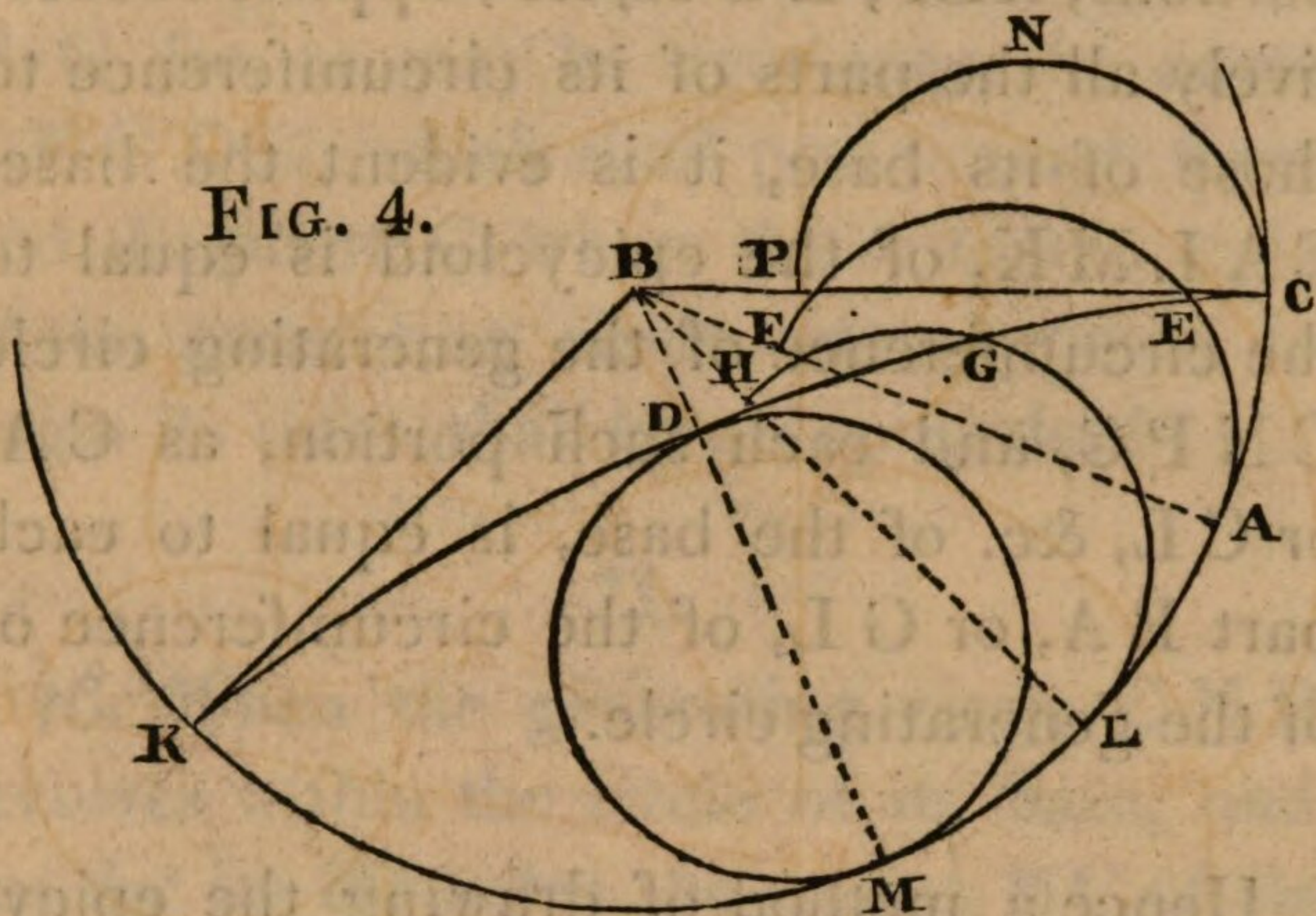


FIG. 3.

II.

13. The circle CNP, which, in revolving describes the epicycloid, is called the *generating circle of the epicycloid*, and the arc CALMK of the immoveable circle, upon which the generating circle revolves, is called the *base of the epicycloid*.

FIG. 4.



III.

14. When the generating circle revolves without the circle of its base, as in Fig. 3. the epicycloid is called an *exterior epicycloid*; and when the generating circle rolls within the circle of its base, as in Fig. 4. the epicycloid is called an *interior epicycloid*.

COROLLARIES.

I.

15. As the generating circle in revolving from its first situation, C N P, to different

portions, AEF, LGH, &c; applies successively all the parts of its circumference to those of its base, it is evident the base, CALMK, of the epicycloid is equal to the circumference of the generating circle CNPC, and each such portion, as CA, or CL, &c. of the base, is equal to each part EA, or GL, of the circumference of the generating circle.

Hence a method of drawing the epicycloid, by describing the circles AEF, LGH, &c. which have all the same radii as the generating circle CNP, and touch the base CALMK in any points A, L, &c; and by making the length of the arcs AE, LG, &c., taken from the points of contact with the base, equal to the arcs AC, CL, &c*.

* In practice, this is most easily done, and with sufficient accuracy, by dividing each arc of the base, as at AC, into a number of small equal parts, and by setting off the same number upon each arc of the generating circle, as at AE.

Having thus determined as many points, E, G, &c. as may be necessary, the curve C E G D K, which shall pass through them and the point C, where the supposed style of the generating circle was supposed to begin its tract, shall be an epicycloid*.

II.

16. When the generating circle C N P revolves within the circle of its base, and has for its diameter the radius B C of its base, the point C, the place of the style during the revolution of the generating circle, will always continue in the diameter C B K. *Hence the epicycloid described by the style C is a straight line, and a dia-*

* To draw the epicycloid mechanically, make the circle of the base and the generating circle of wood, and having fixed a tracer in the circumference of the generating circle, let the base remain at rest, and the tracer, during the rolling of the generating circle, will draw an epicycloid.—In order to make circles move with more accuracy, a small piece of tape may have one of its ends nailed to the circumference of the one circle, and the other end to the other circle.

meter of the circle of its base *; and the circumference of the generating circle C N P being half that of the base, the commencement C and termination K of the epicycloid, must divide the circumference of the base into two equal parts, and the diameter A B of the generating circle being half that of K C of the base, when the generating circle is in the middle of its progress, the point C must be in the centre of the circle of the base; hence we have a point at the origin, in the middle, and at the end of the epicycloid, which all lie in the diameter of the base, and the whole epicycloid may be considered as coinciding with the diameter of the base.†

* Upon this principle a parallel motion has been constructed. It is used by Messrs. Fenton, Murray, and Wood, in some of their smaller steam engines.

For a short account of it, see Gregory's Mechanics, vol. ii. p. 265.

† It would carry me too far into mathematics for many readers, where I strictly to demonstrate, that every point of the epicycloid must lie in the diameter of the base; what is said however will, I hope, satisfy

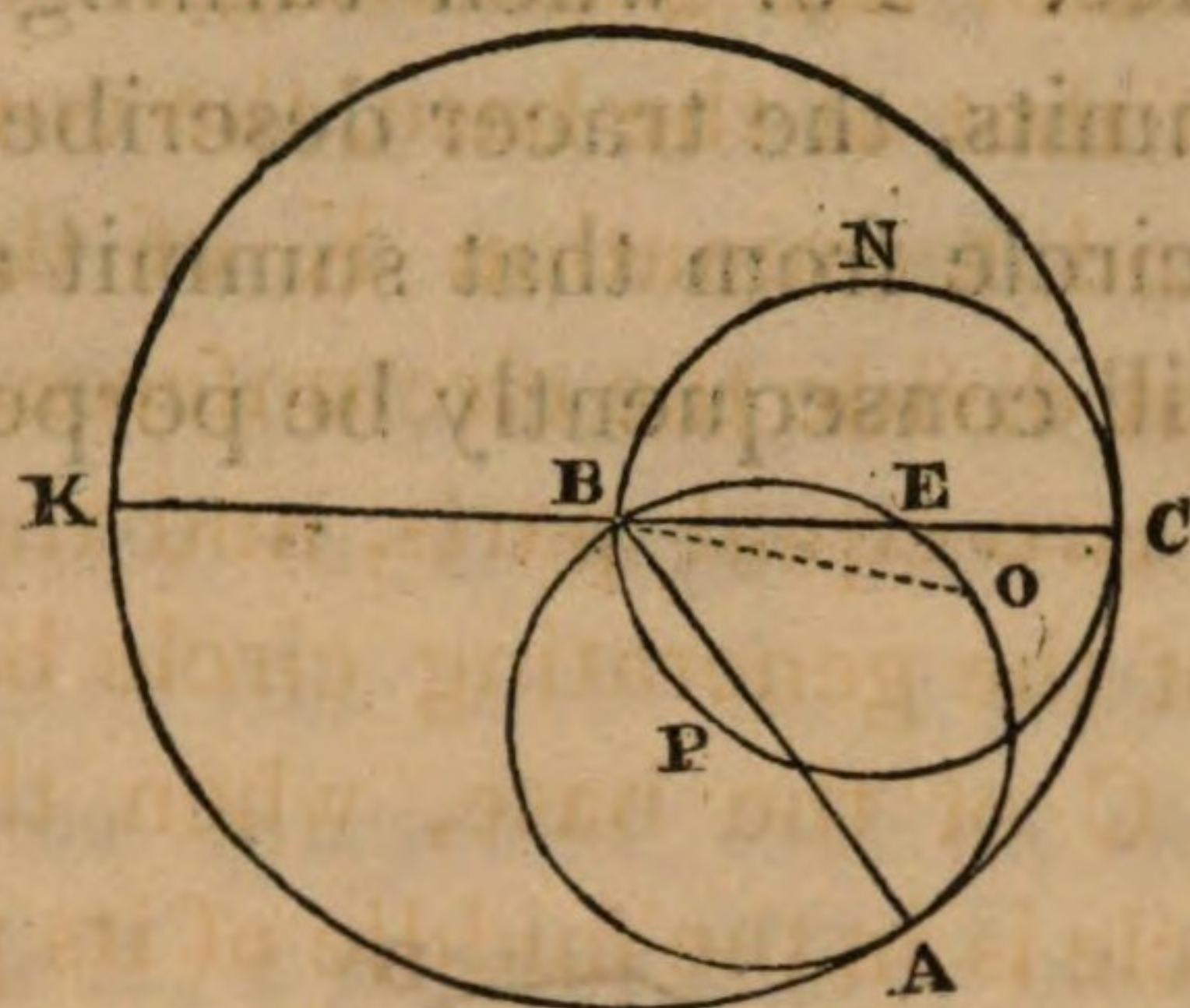


FIG. 5.

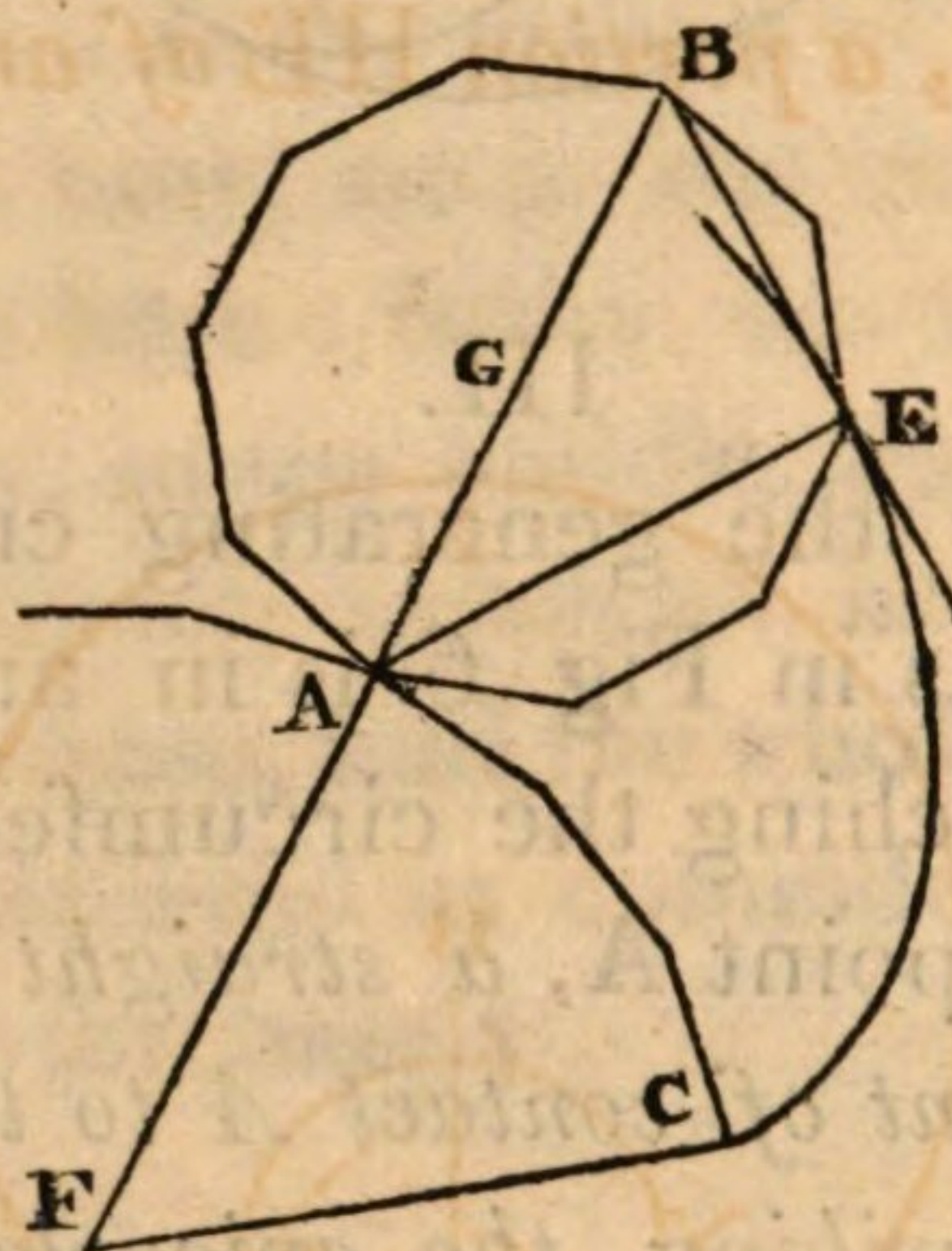
III.

17. When the generating circle of the epicycloid, as in Fig. 6. is in any position, A, E, B, touching the circumference of its base in any point A, a *straight line, drawn from the point of contact A to the point E, actually describing the epicycloid, will be perpendicular to it.*

This will be evident by supposing the them of the truth. The mathematical reader will find a demonstration of it in "Cours de Mathematique, par Camus," iv. No. 538; or English Translation of that part which treats of the Teeth of Wheels, p. 24.

circles to be polygons, having a great number of sides. For when turning on any of the summits, the tracer describes a small part of a circle from that summit as a centre, and will consequently be perpendicular to it.

FIG. 6.

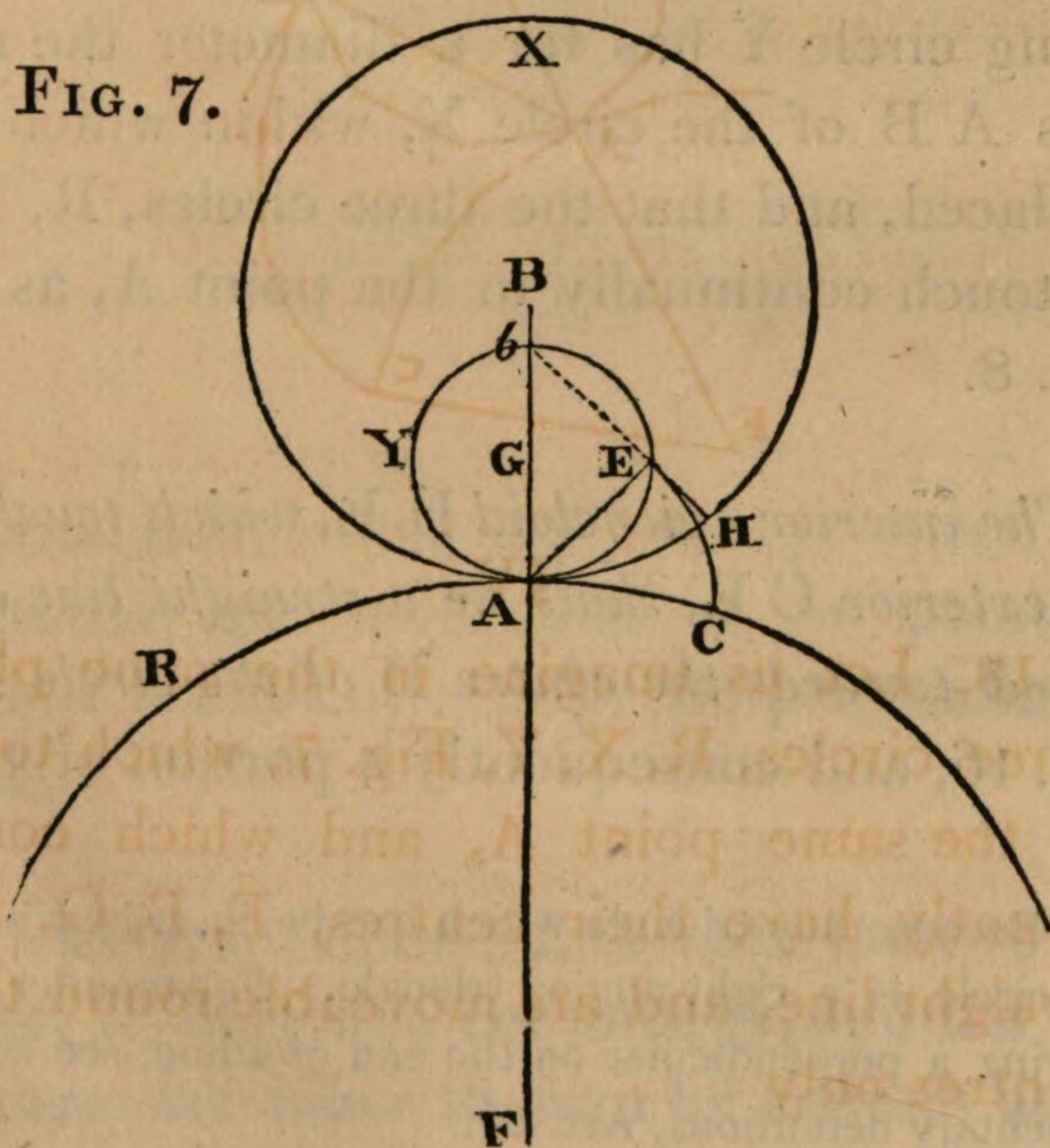


IV.

18. Let us imagine in the same plane three circles, R, X, Y, Fig. 7, which touch in the same point A, and which consequently have their centres, F, B, G, in a straight line, and are moveable round their centres only.

Suppose a style fixed in the circumference of the circle Y, and that the three circles are made to turn by the movement of one of them: if we make each of the arcs, A H, A C, equal to A E, then the style placed in E shall have described on the plane of the circle R, a portion C E of an exterior epicycloid, and on the plane of the circle X, a portion H E of an interior epicycloid.

FIG. 7.



*The two epicycloids, C E, H E, traced in the same time by the style E, touch in the point E. For the straight line A E, drawn from the point A, where the generating circle Y touches its base R C, shall be perpendicular to the two epicycloids, and the straight line H E shall touch the epicycloid in the point E.**

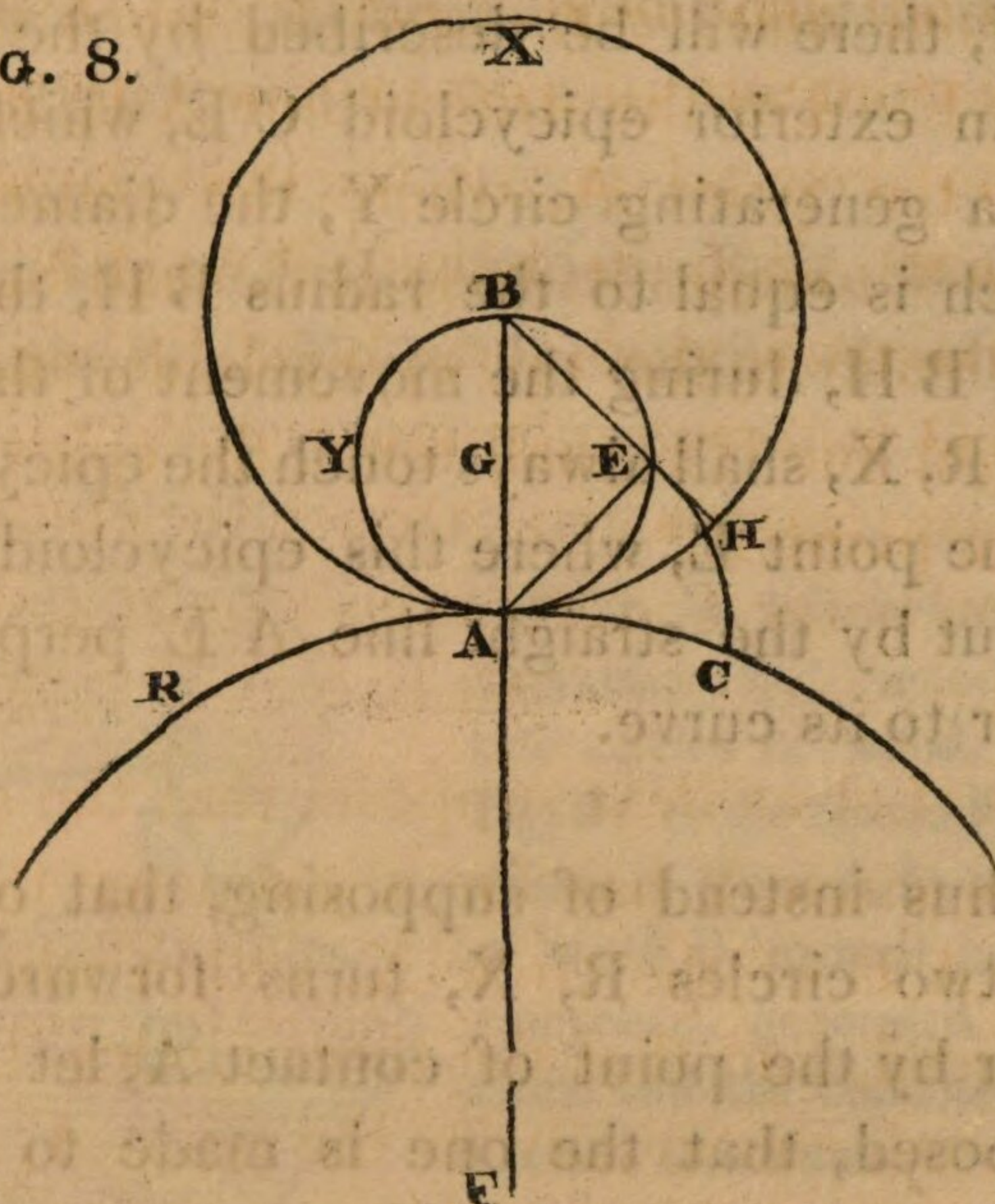
V.

19. Let us next suppose, that the generating circle Y has for a diameter the radius A B of the circle X, within which it is placed, and that the three circles, R, X, Y, touch continually in the point A, as in Fig. 8.

The interior epicycloid H E, which touches the exterior C E, shall be a straight line directed towards the centre B of the circle X, Art. 16, and consequently a portion of the

* Because any triangle which can be inscribed in a semicircle, is a right-angled triangle. For manner of drawing a perpendicular on the end of a line, see supplementary definitions, Art. 65.

FIG. 8.



radius BH , which shall always touch the exterior epicycloid CE in the point E , where it shall be met by the perpendicular AE .

Hence it follows, that when the two circles, R, X , touch continually, and the one causes the other to turn by contact at the point A , if we imagine a radius BH in the

circle X ; and having made AC equal to AH , there will be described by the point C , an exterior epicycloid CE , which has for a generating circle Y , the diameter of which is equal to the radius BH , this radius BH , during the movement of the circles R, X , shall always touch the epicycloid in the point E , where this epicycloid shall be cut by the straight line AE perpendicular to its curve.

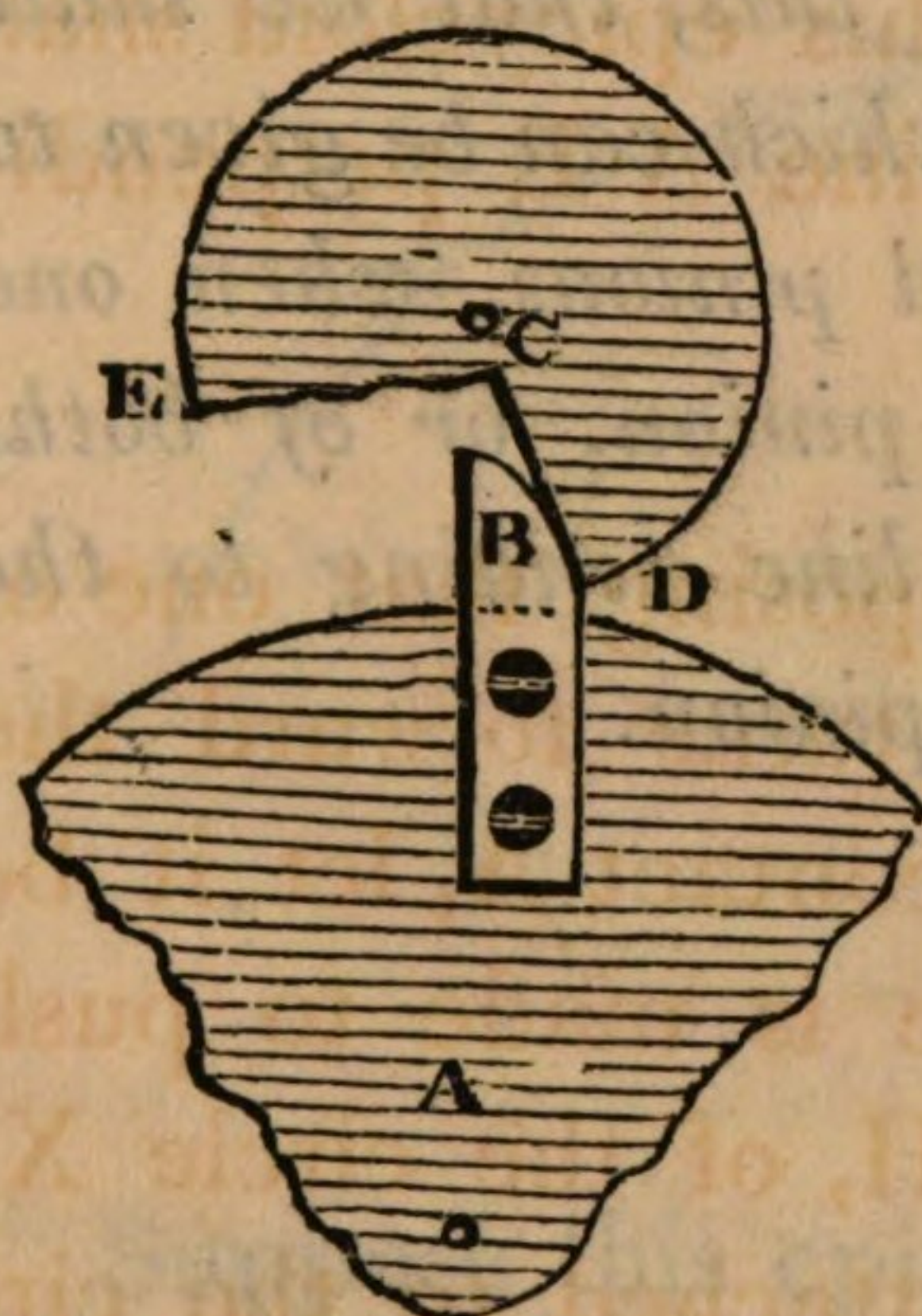
Thus instead of supposing, that one of the two circles R, X , turns forward the other by the point of contact A , let it be supposed, that the one is made to push forward the radius BH , of the circle X , by an epicycloid CE attached to the circle R , and described by the movement of the circle Y , the diameter of which is equal to the radius BH .

One may be able thus reciprocally to make the epicycloid CE , attached to the circle R , push forward by a radius BH a circle X ; and by means of the epicycloid CE , and of the radius, BH , the two cir-

cles, R, X, may be able to conduct themselves as if put forward by the point of contact A*.

For suppose the radius, B H, and the epicycloid, C E, to be teeth of wheels,

FIG. 9.



* To be satisfied of this experimentally, make any two circles of wood, as in Fig. 9; to the circumference of one of them A, fix a piece of wood B, formed into an epicycloid, generated by a circle half the diameter of C upon A as a base.

On the circle C, draw the line C D, and cut out the part bounded by that line and C E.

If you cause one of the circles to move the other by the parts B, C D, both circles will have the same velocity; as may be ascertained by putting a mark opposite any point in the circumference of each circle before they begin to move, and another after they stop, and the distance between which, measuring by the arcs, will be found equal.

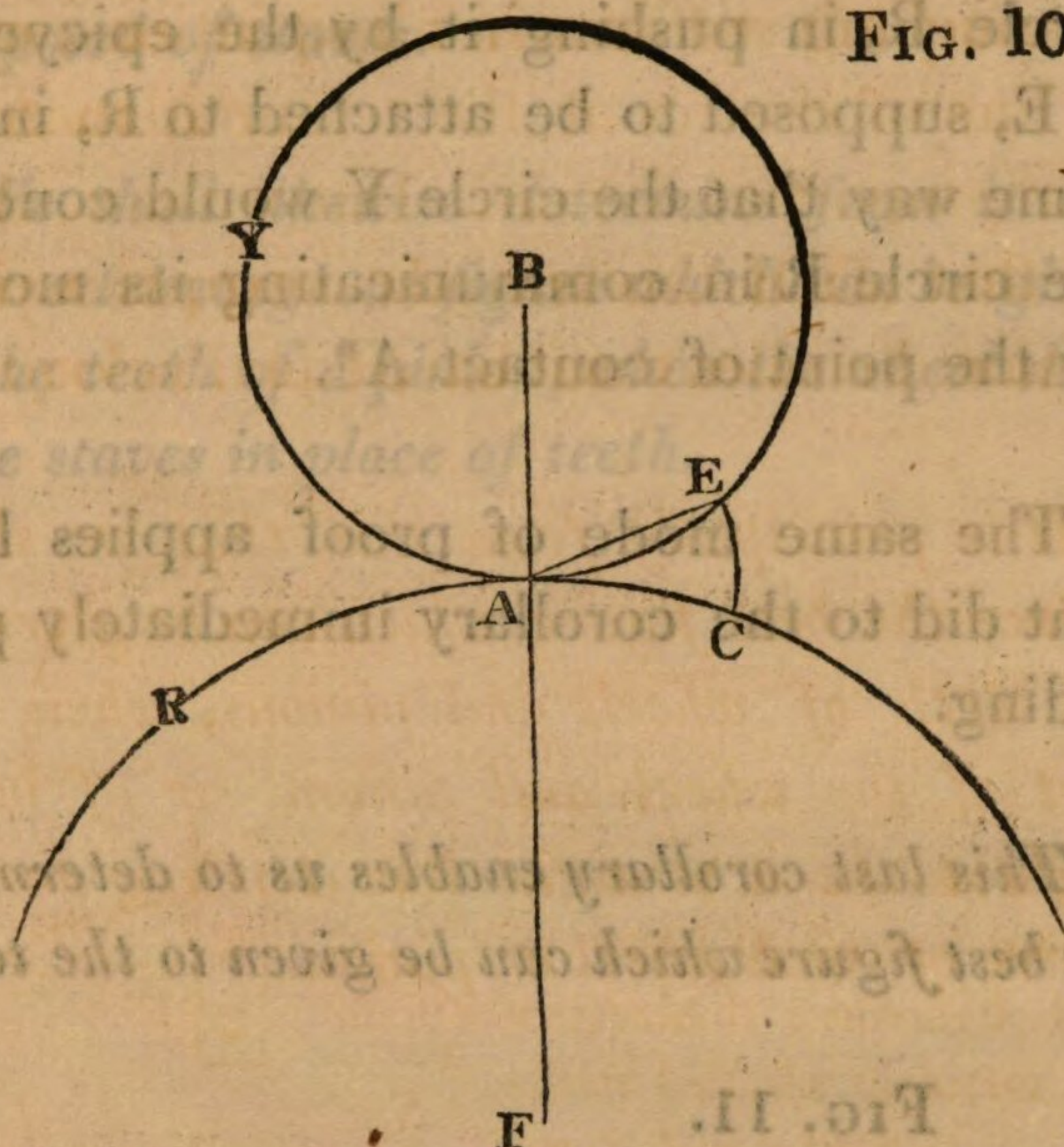
X and Y ; and the perpendicular A E, from the touching surfaces in all situations, cuts the line of centres at the termination A of their proportional radii. But we saw, Art. 11, that when this was the case, the proportional circles must have equal velocities.

It is principally from this, that we shall deduce the best figure which can be given to the teeth of wheels and pinions, when one part of the wheel and pinion, or of both, ought to be a straight line tending to the centre of such wheel or pinion.

VI.

20. If in the same plane we have but two circles, R, Y, Fig. 10. which touch in the point A, and if the movement of the one communicate itself to the other, by this point of contact, any point E of the circumference of the circle Y, describes upon the plane of the moveable circle R, an epicycloid C E.

FIG. 10.



Suppose this epicycloid attached to the circle R, it (the epicycloid) shall conduct the circle Y, pushing it round by the point E of its circumference, in the same manner as the circle R might conduct the same circle Y in communicating motion to it by the point of contact A.

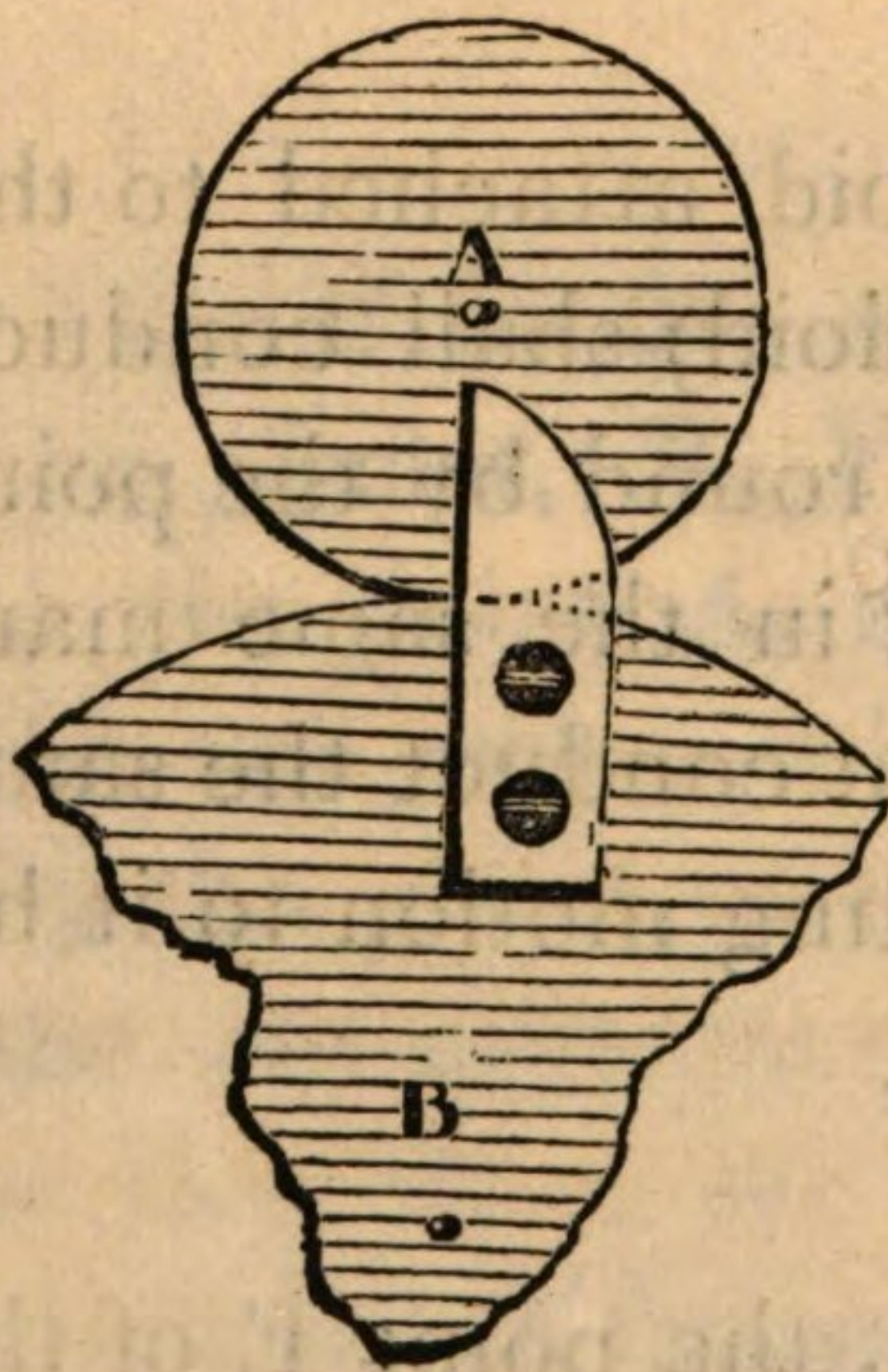
And in like manner, the point E of the

circumference of the circle Y, turns the circle R, in pushing it by the epicycloid C E, supposed to be attached to R, in the same way that the circle Y would conduct the circle R in communicating its motion by the point of contact A*.

The same mode of proof applies here that did to the corollary immediately preceding.

This last corollary enables us to determine the best figure which can be given to the teeth

FIG. 11.

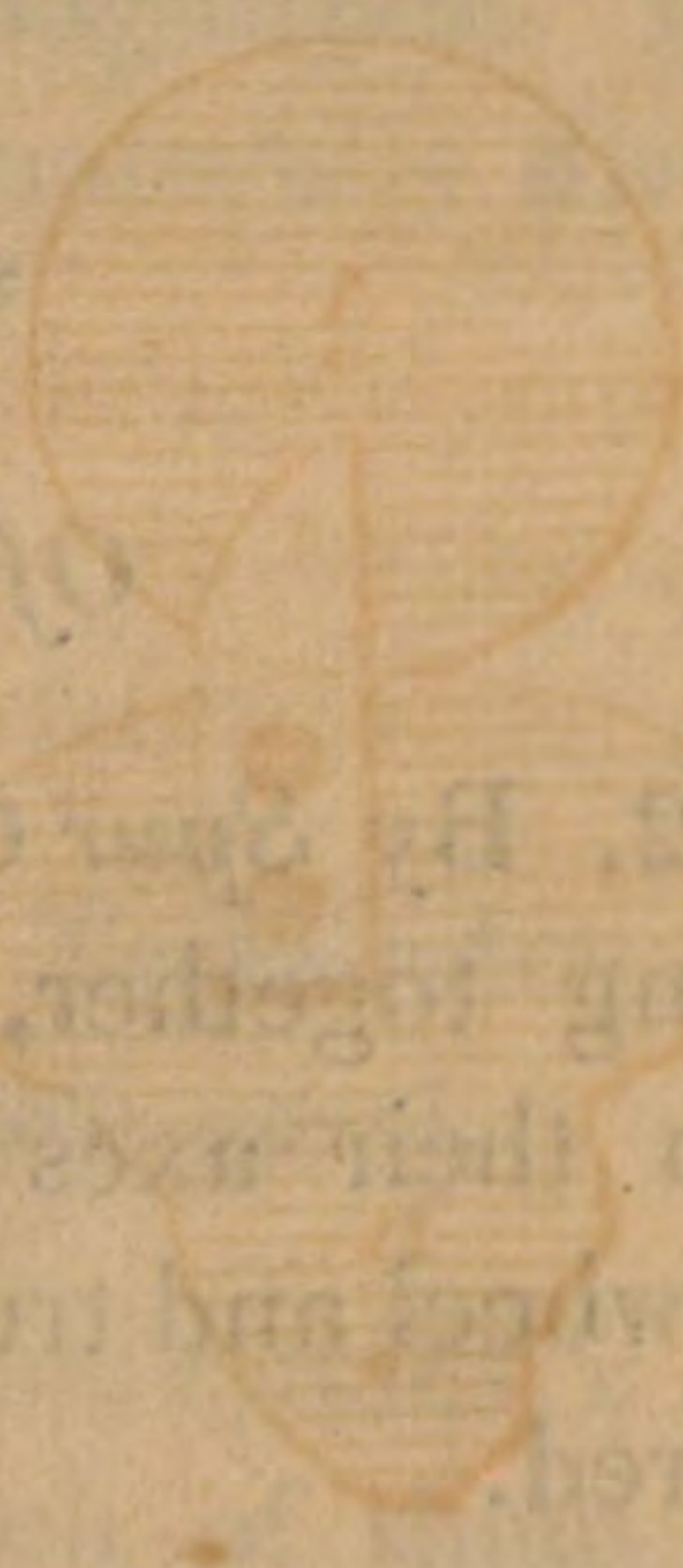


* The experiment to prove this is similar to the former, but with this difference, that in the circumference of one of them, A, is fixed a fine needle, which is made to act against a piece of wood, formed into an epicycloid, fixed upon the other, B, which epicycloid is generated by A upon B as a base.

of wheels, when the pinion shall be a trundle composed of staves.

We shall likewise determine from it the most advantageous figure which can be given to the teeth of a pinion, when the wheel shall have staves in place of teeth.

FIG. 11.



CHAP. II.

OF THE APPLICATION OF THE PRINCIPLES OF THE
CONFIGURATION OF THE TEETH OF WHEELS.

21. Having endeavoured to show, that an epicycloid is a curve, whereby two circles may conduct themselves as if put forward by the simple contact of their circumferences, I shall now attempt a practical explanation of this curve, in giving the best form to the teeth of wheels.

SECTION I.

Of Spur Geers.

22. By *Spur Geers* is understood wheels acting together, and in the same plane, with their axes parallel; under this head the wheel and trundle come first to be considered.

Of the Wheel and Trundle.

23. To determine the figure of the teeth of the wheel, which depends always upon that of the staves of the trundle, we shall first suppose the staves indefinitely small, and represented (Fig. 12) on the end of the trundle by the points A, E, H, &c.: when we have found the figure of the teeth proper to conduct the indefinitely small staves, (which are used for demonstration only,) we shall, by means of that figure, trace the true form which should be given to the teeth of wheels to conduct trundles with cylindric staves of some magnitude. Thus the solution of this case, naturally divides itself into two parts.

I.

To find the Figure of the Teeth when the Staves are indefinitely small.

24. Draw the proportional circles, c A C and e A E, and divide each of them into

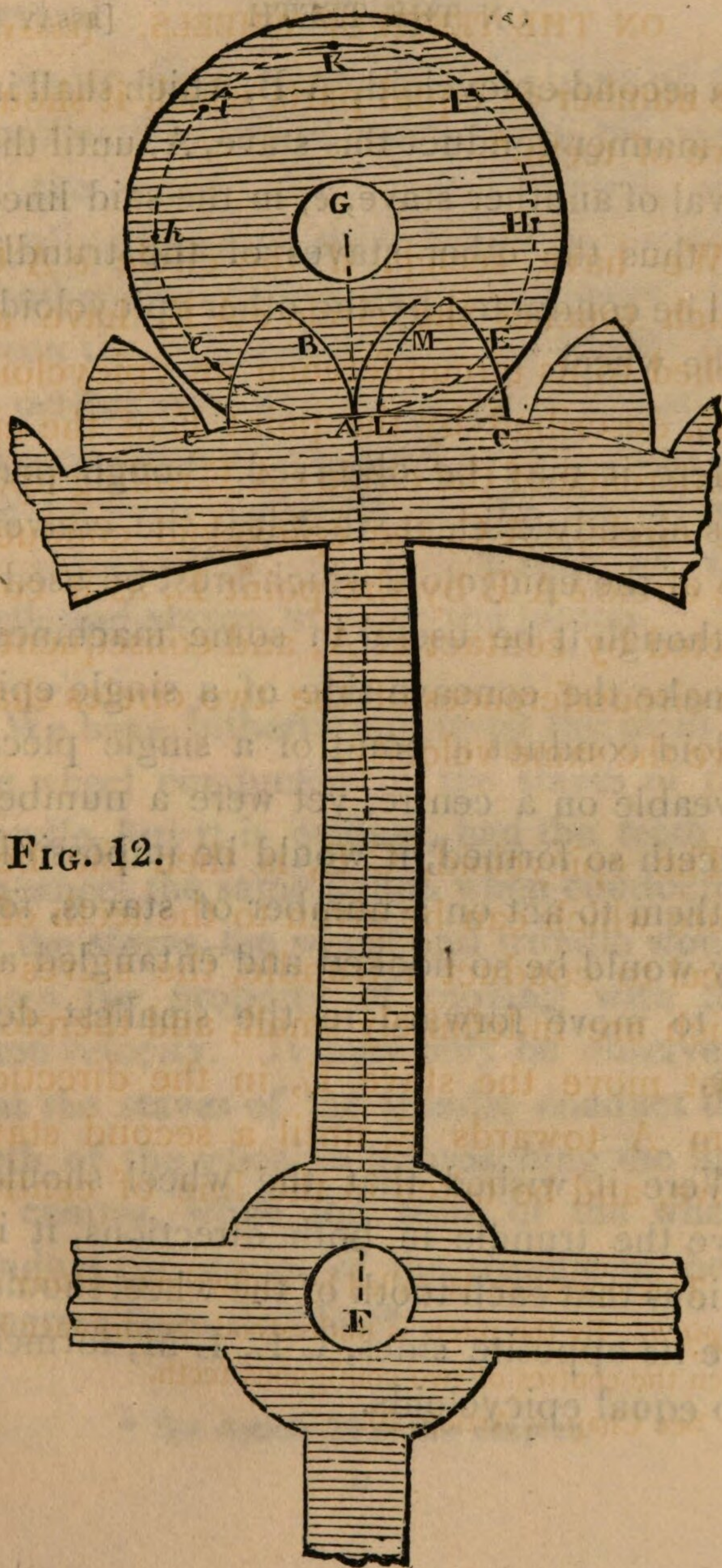
the number of equal parts which it should have of teeth.*

We have seen†, if the circle $c A C$, which touches the circle $e A E$, have attached to its circumference an epicycloid, $C E$, described by the point E of the circumference of the circle $e A E$ rolling upon the circle, $c A C$, the epicycloid conducts the circle $e A E$ by the point E , as if conducted by contact at A , and consequently the circumferences of the two circles shall have the same velocity.

The epicycloid, $C E$, is then the best figure which can be given to the teeth of a wheel to conduct a trundle, the staves of which are indefinitely small, and therefore must move the stave E , in the direction from A towards E , until a second stave arrive, and be taken in the line of centres

* This operation is called by millwrights *setting off the pitch*. By the pitch is understood the distance between the centres of two contiguous teeth.

† See Chap. i. Article 20.



by a second epicycloid, A B, which shall in like manner conduct this stave, A, until the arrival of another stave, *e*, in the said line : and thus the other staves of the trundle shall be conducted by the other epicycloids of the wheel.

Here it may be observed, though perhaps already evident, that it is the convex side of the epicycloid which must be used : for though it be useful in some machines, to make the concave side of a single epicycloid conduct a point of a single piece moveable on a centre, yet were a number of teeth so formed, it would be impossible for them to act on a number of staves, for they would be so hooked and entangled as not to move forward in the smallest degree.

Were it wished that the wheel should move the trundle in both directions, it is obvious that each tooth of the wheel should have its opposite sides, C E, L M, formed into equal epicycloids.

As we have supposed the staves of the trundle indefinitely small, were the teeth of the wheel also perfect figures, and equally distanced, there would be no need of other than indefinitely small spaces between the adjacent teeth of the wheel; but as perfect precision is not to be expected, a space more or less, such as A L, must be left between them, to enable the wheel, notwithstanding the inequalities of the teeth and staves, to move the pinion.

We have hitherto supposed the teeth of the wheel conducted by the staves of the trundle, but it is evident, had the teeth of the wheel the same figure, when conducted by the staves, the wheel and trundle would retain the property of moving with the same velocity. It may only be observed, that the staves of the trundle conduct the teeth of the wheel in approaching the line of centres, while the teeth of the wheel conduct the staves of the trundle in their progress from that line*.

* See Article 28 of this chapter.

II.

To find the figures of the teeth of the wheel, when the staves of the trundle are cylinders of a finite diameter.

25. Consider the trundle at first as having infinitely small staves, represented by the centres of the staves, A, E, H, &c. and trace, as above mentioned, the teeth C L P, A Q N, &c. of the wheel, as if it had to conduct a trundle with infinitely small staves: observing to leave a small space, such as A L, between all the teeth, in order that they may act freely.

Describe, with the radius of the staves, upon the plane of each tooth, as many small arcs as may be convenient, having all their centres in the two epicycloids which form the teeth.

Trace, by means of these little arcs, two curves, such as R O, S O, parallel to the epicycloids, and then you will have inclosed the space, R O S, which is the figure

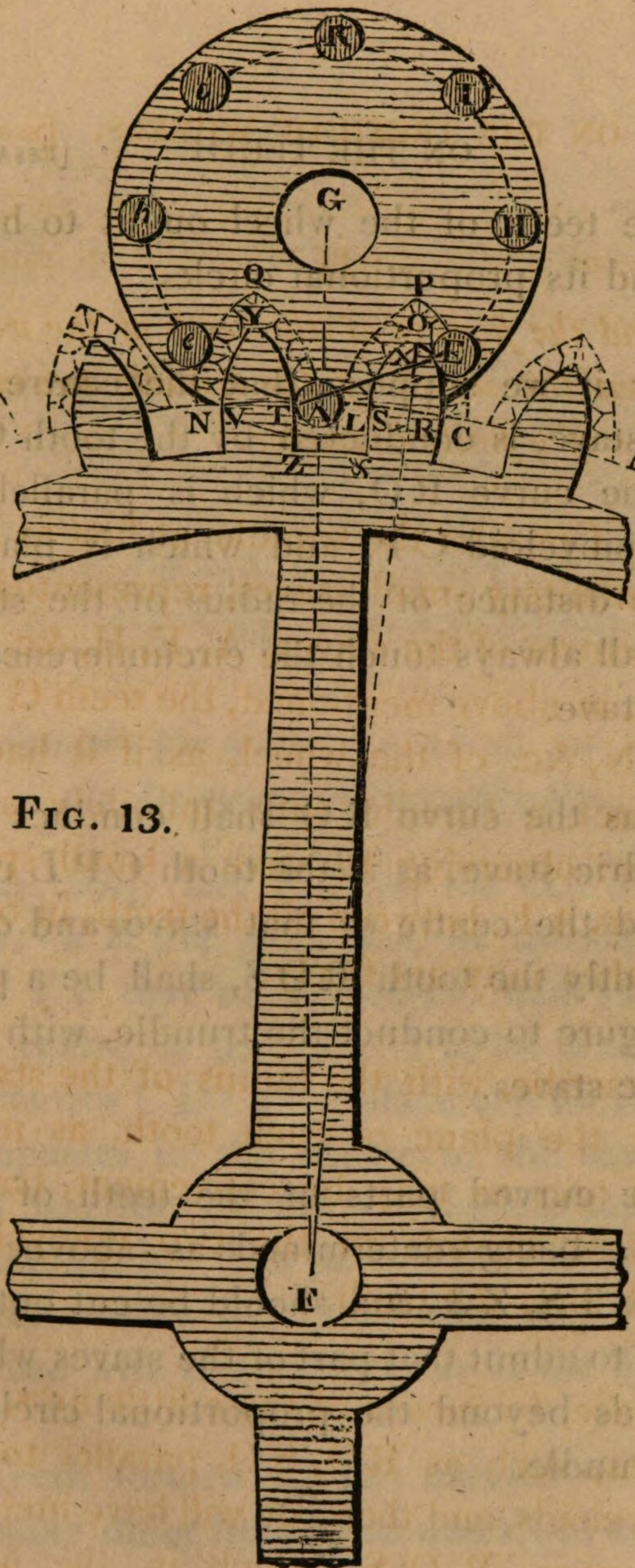


FIG. 13.

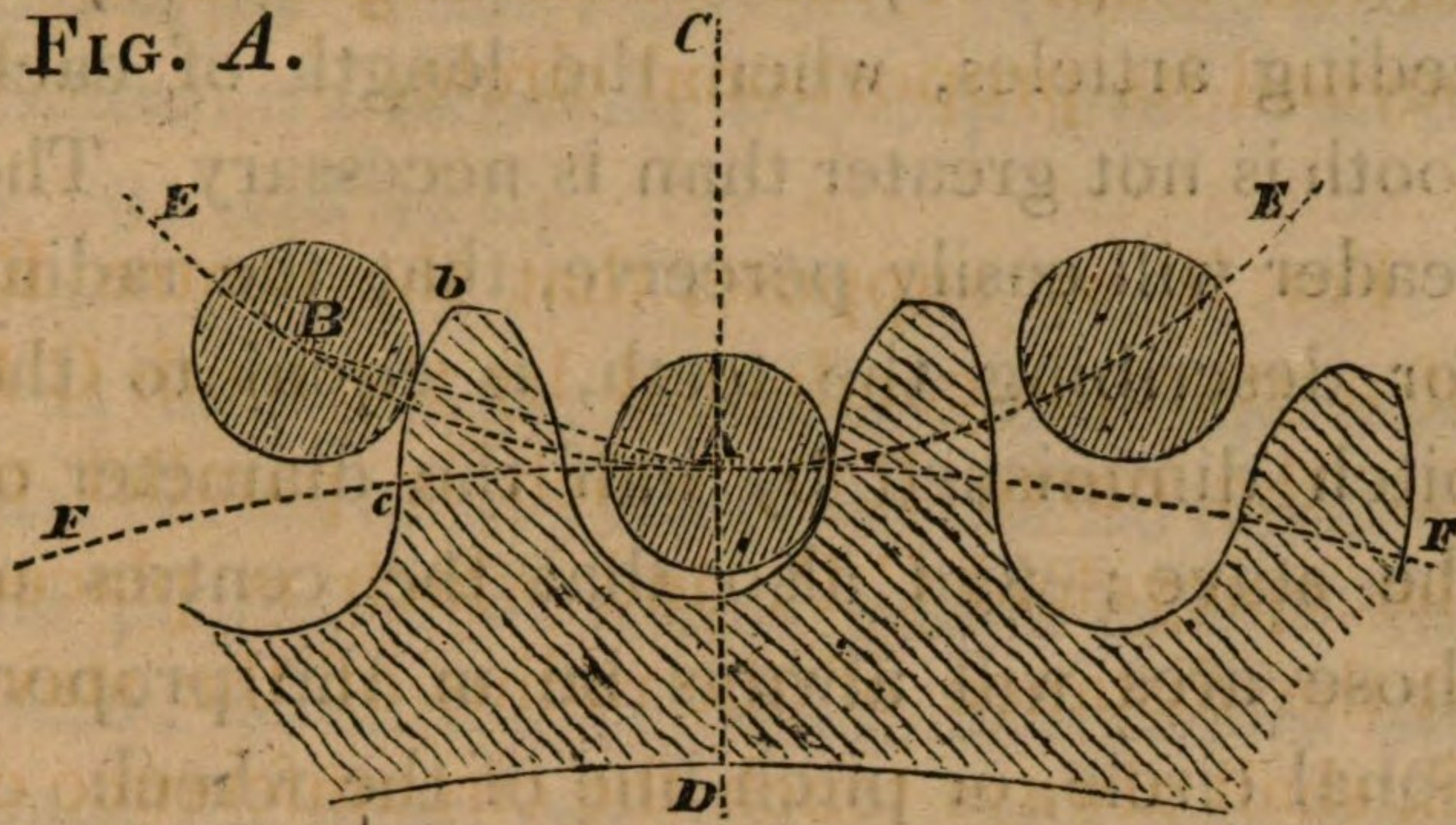
all the teeth of the wheel ought to have beyond its proportional circle.

For if we suppose, that the centre E, of a stave, is conducted by the tooth C P L; the curve R O, which is parallel to the epicycloid C P, and which is placed at the distance of the radius of the stave E, shall always touch the circumference of that stave.

Thus the curve R O shall conduct the cylindric stave, as if the tooth C P L conducted the centre of that stave, and consequently the tooth R O S, shall be a proper figure to conduct the trundle, with cylindric staves.

The curved parts of the teeth of the wheel, being determined as above, the spaces T S, Z &, &c. should be cut out, in order to admit that part of the staves which extends beyond the proportional circle of the trundle.

FIG. A.



To describe the teeth of a wheel, for a trundle, by means of circular arcs.

[A. 25.] Let C D be the line of centres; E E the pitch line of the trundle; and F F that of the wheel; and suppose the centre of the stave A to be in the line of centres C D: then place one foot of the compasses in the centre of the stave A, and describes the arc *b c*, which is the form of the tooth. The part of the teeth of the wheel, within the pitch line, may be described with circular arcs as in the figure.

Teeth formed in this manner will not sensibly differ from those described accord.

ing to the principles laid down in the preceding articles, when the length of each tooth is not greater than is necessary. The reader will easily perceive, that the radius for describing the teeth, is equal to the pitch diminished by half the diameter of the stave; and also that the centres of those arcs will always be in the proportional circle, or pitch line of the wheel.

Some authors have imagined that the friction of the wheel and trundle might be reduced by making the staves revolve; but it could not be effected so far as to balance the extra labour of construction, and where the strain would be considerable, it would become quite impracticable. (See Emerson's *Mechanics*, prop. 119, rule 9; and *Transactions of the Society of Arts*, vol. xxxv. p. 128.)

Smeaton appears to have been very partial to the wheel and trundle, when the trundle was executed with cast iron staves; these he recommended to be of an oval

figure, and made smooth by grinding them. (Smeaton's Reports, vol. i. p. 316; vol. ii. p. 391 and 423.)

It may be demonstrated that the least real radius of the wheel should be equal to the proportional radius added to half the pitch; when the necessary allowances are made for wear, (see Art. B. 32.) and when the staves are of the same diameter as the thickness of the teeth. But when the staves are larger than the teeth, as in the figure, a less real radius is required. (Ed.)

Having considered the case of a wheel and trundle, with cylindric staves acting together, I am now to explain that of a wheel and pinion, two sides of the figure of whose teeth are straight lines directed to its centre.

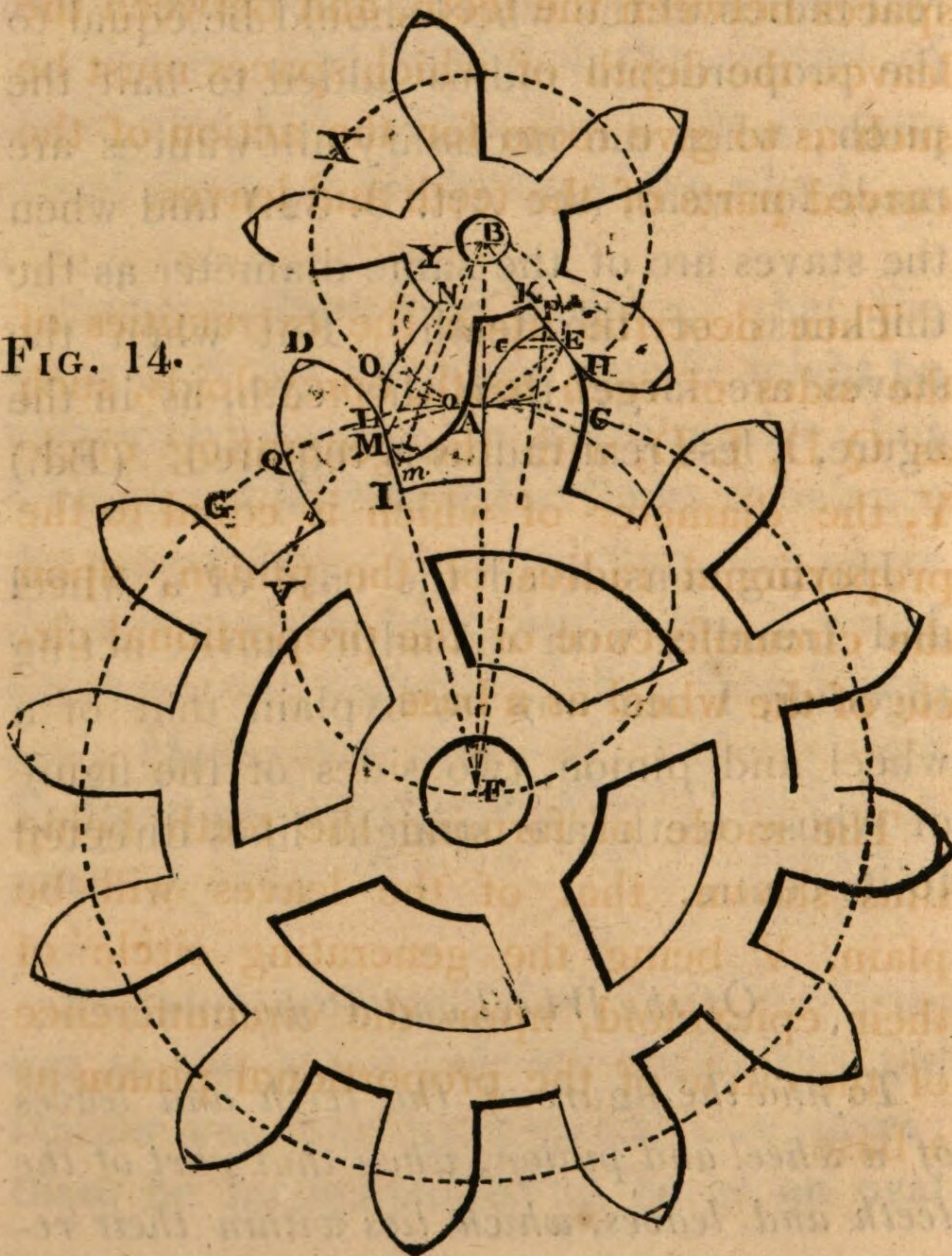
Of the Wheel and Pinion.

To find the figure of the teeth and leaves of a wheel and pinion, when that part of the teeth and leaves, which lies within their re-

spective proportional circles are straight lines directed to the centres of these circles.

26. Having set off upon the proportional

FIG. 14.



circles, the points, G, Q, L, and, O, o, H, &c. according to the thickness of the teeth and leaves, draw lines from these points, tending towards the centre of their respective circles, to serve as the sides of the spaces between the teeth, and between the leaves, the depth of which spaces must be such as to give room for the action of the curved parts of the teeth and leaves.

Then describe upon the extremities of the sides of each tooth, epicycloids, such as Q D, L D, with the generating circle Y, the diameter of which is equal to the proportional radius of the pinion, upon the circumference of the proportional circle of the wheel as a base.

The mode of forming the teeth being thus shown, that of the leaves will be plain, V being the generating circle of their epicycloid, upon the circumference of the circle of the proportional pinion as a base.

We have seen*, if the radius BH of the proportional pinion, be pushed by an epicycloid CP , generated by the circle Y , upon the pitch line of the wheel, and projecting therefrom, the pinion shall turn with the same velocity as the wheel.

In the same manner it may be proved, that the same effect will be produced, if the epicycloid OMm , attached to the pinion, be pushed towards the line of centres, by the radius, LF , of the wheel.

Lastly, the two opposite sides of the teeth, and those of the leaves, ought to have the same figure, for the ease of action, and to give the wheel and pinion the liberty of being moved in either direction.

From these principles, I hope, it will be evident, that the figure here given to the teeth, will make the wheel and pinion move with perfect regularity.

* Chap. I. Article 19.

[A. 26.] To make that part of a tooth which is within the pitch line or proportional circle a straight line, as proposed by the Author, seems to be the most advantageous form, because it causes least pressure on the axes.

When the teeth are small, and do not begin to act till they arrive at the line of centres, the teeth of the wheel, when the wheel drives the pinion, or the leaves of the pinion, when the pinion drives the wheel, may be described by a circular arc of which the radius is equal to the pitch; and of which the centre is in the pitch line of the wheel or pinion.

This method will always enable a workman to execute short teeth nearer to the true form than any pattern tooth will enable him to do. Pattern teeth and compound curves, are things that may on some occasions be very useful; where the teeth are long, and of considerable magnitude in respect to that of the wheel or pinion to

which they belong. But in all the ordinary forms of wheel-work such operations must consume an immense quantity of valuable labour to attain even the same degree of accuracy that is at once obtained by means of circular arcs. When a pattern tooth is necessary, one of its adjustments should be the centre of the wheel; and not two points in its circumference, as proposed in Imison's Elements of Science and Art, (vol. i. p. 103.) because the latter method at least doubles the risk of error in adjusting the pattern.

When part of the action takes place before the teeth arrive at the line of centres, the method of forming teeth proposed by our Author (Art. 32.) seems to be equal, if not superior, to any other. And its practical application is shown in Art. A 32. (Ed.)

REMARKS.

27. As it is the curved part of the teeth of the wheel, that should push the straight flank H K, of those of the pinion, in re-

moving from the line of centres, and as the point E, where the flank is acted upon, is a perpendicular drawn from A, it shall be always that by which the wheel shall push, it is clear, that when the extremity P, of the epicycloid C P, reaches the point E, it shall cease to move the tooth H K; if the extremity P, arrive at the point E, before the flank O N, of the following tooth of the pinion has reached the line of centres, the curved part, O, M, m, of this tooth, must be pushed by the straight flank L I, of the following tooth of the wheel, till the flank O N, reaches that line: so that in this case, the wheel conducts the pinion, at one time before, and, at another, beyond the line of centres.

But were it so, that the extremity P, did not reach the point E, till after the flank, O N, had arrived at the line of centres; it would not be necessary, that the curved parts of the leaves, should be pushed by the flanks of the teeth. Thus, in this case, the wheel would conduct the pinion, in

pushing its leaves beyond the line of centres only.

28. It is the general opinion of those who are in the practice of constructing wheel work, that teeth ought, if possible, never to begin to act before they reach the line of centres, as that mode of action is thought to occasion much unnecessary friction.* The cause of this great unnecessary friction, when the teeth are of wood, appears to be the following:

Friction depends not only upon the

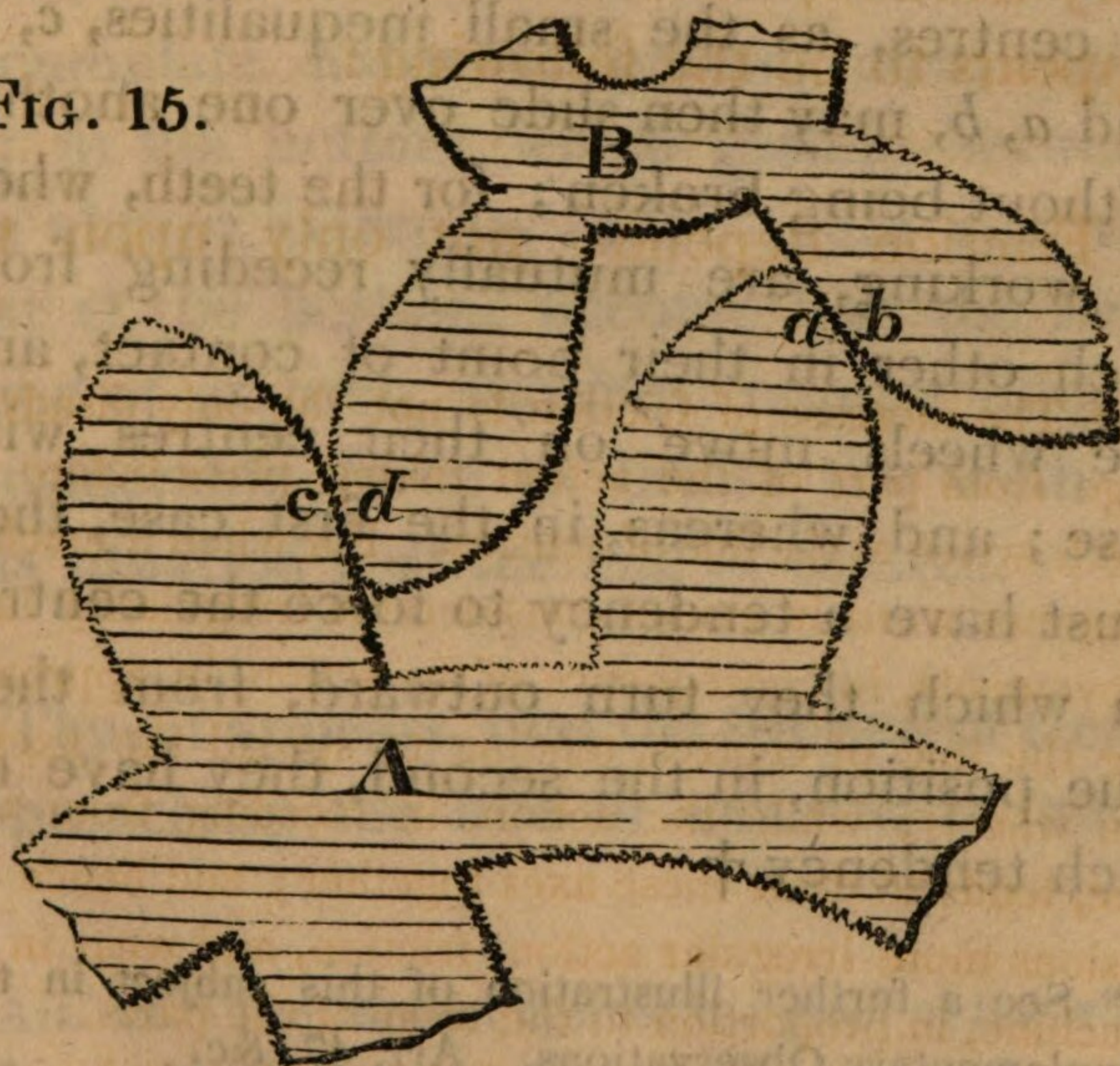
* The increase of friction is not the only disadvantage from part of the action taking place before the teeth arrive at the line of centres. For when a machine becomes worn it causes considerable irregularity in its movements, in consequence of the action, in approaching the line of centres, tending to spread the axes of the wheels; while the action, in receding from that line, tends to draw these axes together; and hence occasions more irregular action, friction, and wear in the machine, in proportion to the wear of the parts.

We have an obvious remedy in making pinions sufficiently large to allow of the action being wholly on one side of the line of centres. (Ed.)

pressure made on moving bodies, but on the inequalities of the surface upon which they move ; and as the surfaces even of the most highly polished bodies have some inequalities, whenever two of them are pressed together, the inequalities of the one must enter the other.

Suppose A and B to be a wheel and

FIG. 15.



pinion, having wooden teeth, as they would appear through a microscope ; it is impossible, though there be no other resistance than that arising from friction, to move them towards the line of centres, until either the centres, on which the wheels turn, give way, or some of the small inequalities, *c, d*, of the teeth be broken off.*

On the other hand, a very small force will move the teeth outwards from the line of centres, as the small inequalities, *c, d*, and *a, b*, may then slide over one another without being broken ; for the teeth, when so working, are mutually receding from each other in their point of contact, and the wheels move on their centres with ease ; and whereas, in the first case, they must have a tendency to force the centres on which they turn outward, from their true position, in the second, they have no such tendency.†

* See a further illustration of this subject in the Supplementary Observations. Art. 47, &c.

† The true cause of the increase of friction is shown

Again, when the teeth are of metal, this unnecessary friction seems to arise principally after the teeth are in some degree worn (See Fig. 18, in p. 60.) The teeth in that case have a kind of seat formed at their bottom, and the curve at the outward extremity is too much inclined to the radius, and very abrupt. In the action which takes place before the line of centres, the sliding of the teeth of the conducting wheel along those of the conducted, has a tendency to accumulate hardened grease, dust, sand, &c. at the bottom, which getting between the abrupt extremity of the tooth and the seat at the bottom, become like the keystone of an arch, and must require often a considerable force to bruise the teeth in this situation past the line of centres.

Thus it appears, that the friction of teeth, approaching the line of centres, is much

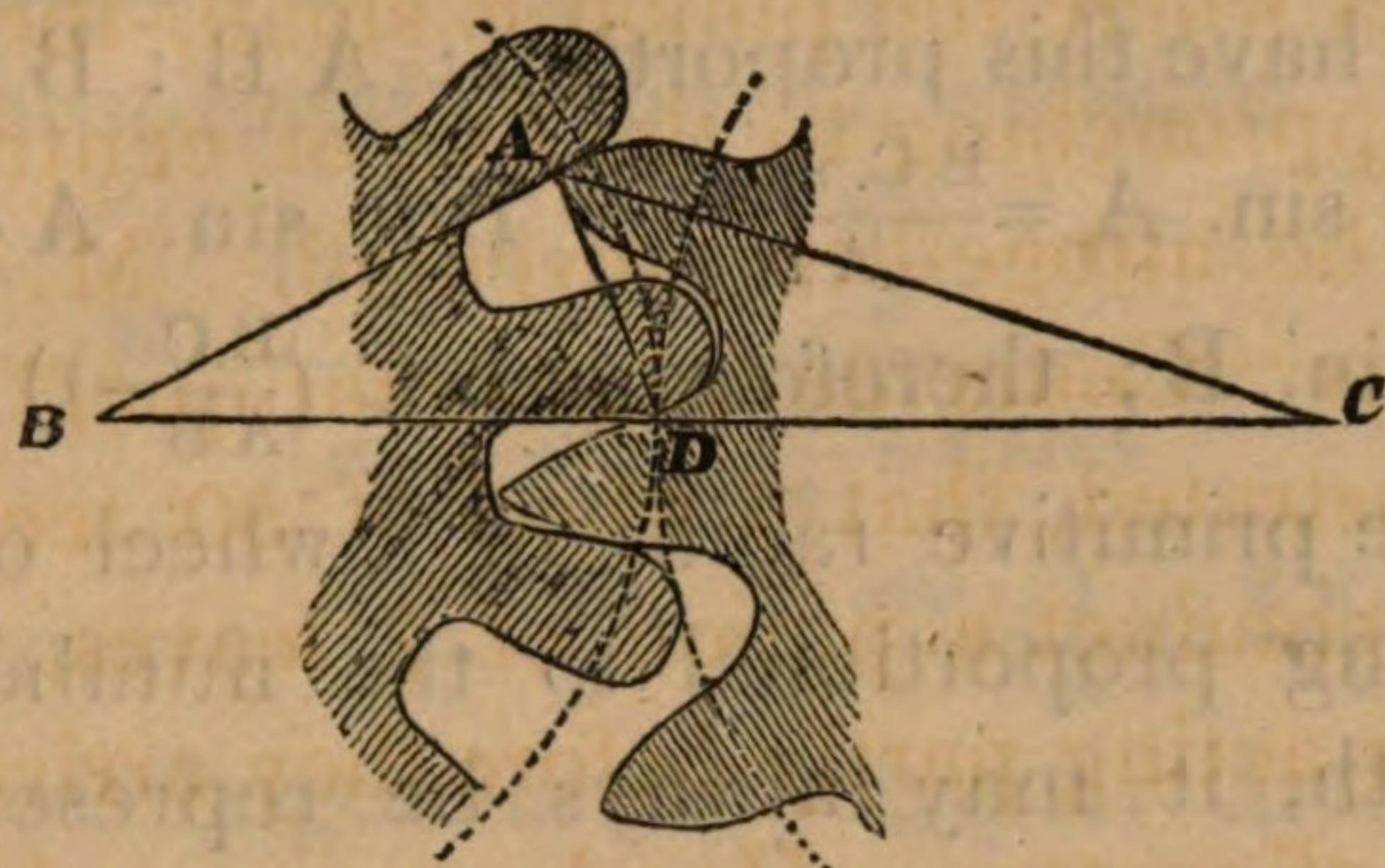
in Art. 48. The motion against the grain of the wood will scarcely have a sensible effect, except when the teeth are new. (ED.)

greater than in receding from it. But in cases where the pinion is small, the action in approaching to the line of centres, cannot be altogether prevented. M. Camus, in his "*Cours de Mathematique*," has demonstrated, that a wheel of 50 teeth cannot conduct a pinion of 7 leaves, without their acting partly before they arrive in the line of centres. He also proves the same with regard to 57 teeth and 8 leaves, 64 and 9, 72 and 10.*

[A. 28.] Since the calculations which Camus has given are confined to particular cases, and troublesome to apply to new ones, because his method is indirect; I will in these additional articles, investigate a rule, which will be found somewhat more general, and of easier application.

* See Camus on the Teeth of Wheels. English translation, p. 60—67.

FIG. B.



Let B be the centre of the pinion, and C the centre of the wheel; then we have to ascertain the relation between the number of teeth on the pinion and the number of those on the wheel, so that any tooth D of the wheel may arrive at the line of centres before the preceding tooth of the wheel quits the tooth A of the pinion.

The last point of contact cannot be beyond the middle point of the tooth, and therefore the line A C will be in the middle of it, A being the point of contact. Also A D will be perpendicular to B A, the point D being in the pitch lines.

Now by the principles of trigonometry we have this proportion ; $A B : B C :: \sin. C : \sin. A = \frac{B C. \sin. C}{A B}$. But $\sin. A - \sin. C = \sin. B$; therefore, $\sin. C (\frac{B C}{A B} - 1) = \sin. B$. The primitive radius of a wheel or pinion being proportional to the number of its teeth, it may always be represented by that number ; and if N be the number of teeth in the wheel, and n those of the pinion, we shall have $B C = N + n$. The line $B A$, being a side of a right-angled triangle, considering the radius unity, is found by this proportion ; $1 : \cos. B :: n : A B = n. \cos. B$. And the angle B is $\frac{360^\circ}{n}$; consequently, $\sin. C (\frac{B C}{A B} - 1) = \sin. B$
 $= \sin. C (\frac{N + n}{n \cos. \frac{360}{n}} - 1) = \sin. \frac{360}{n}$.

[B. 28.] As the $\sin. C$ is, in practical cases, sensibly equal to its corresponding arc, we may use the arc for the sine, which will render the equation much more sim-

ple. The arc is $\frac{2 \times 3.4161}{N} \times \frac{3}{4} = \frac{4.7124}{N}$; which being substituted for sin. C, we have

$$N = \frac{4.7124 n (1 - \cos. \frac{360}{n})}{n \sin. \frac{360}{n} \cdot \cos. \frac{360}{n} - 4.7124}$$

Or, because $\sin. \frac{360}{n} \times \cos. \frac{360}{n} = \frac{1}{2} \sin. \frac{2 \times 360}{n}$.
(Gregory's Trigonometry, Chap. iv. Art. 20.)

$$N = \frac{9.4248 (1 - \cos. \frac{360}{n})}{\sin. \frac{720}{n} - \frac{9.4248}{n}}$$

This equation gives a result nearer to the conditions required in the actual construction of wheels and pinions than when the sine is used instead of the arc; for it makes an allowance for the wearing away of the teeth. It supposes the teeth of the pinion to be the same size as the teeth of the wheel, which is the least thickness that can be given consistent with strength and durability.

[C. 28.] The above equation shows, that when a pinion has less than 10 leaves, it cannot be conducted uniformly by a wheel

with any number of teeth whatever, unless they act partly before they arrive at the line of centres ; because the denominator of the second member of the equation becomes negative when n is less than 10.

But a pinion of 10 leaves may be moved uniformly by a wheel with 209 teeth, when the whole of the action is after they arrive at the line of centres ; for in this case $n=10$, and by a table of sines, we find $\cos. 36^\circ = .80902$; and $\sin. 72^\circ = .95106$. Therefore
$$\frac{9.4248 (1 - .80902)}{.95106 - .94248} = 209 = N.$$

In like manner it may be calculated that a pinion of 11 leaves may be moved uniformly by a wheel having not less than 28 teeth ; and a pinion of 12 leaves when the wheel has not less than 15 teeth ; so that the whole of the action may be after the teeth arrive at the line of centres.

[D. 28.] The same equation may easily be extended to the case of the wheel and

trundle, and as it does not appear to have been investigated, I shall here show the result under the following conditions ; 1. The extremity of each tooth of the wheel is to be one third of its thickness at the root. 2. The centre of the stave is to be in the pitch line when the point of the tooth quits the preceding stave. When limited to these circumstances the equation in Art. B. 28. becomes

$$N = \frac{7.33 (1 - \cos. \frac{270}{n})}{\sin. \frac{540}{n} - \frac{7.33}{n}}$$

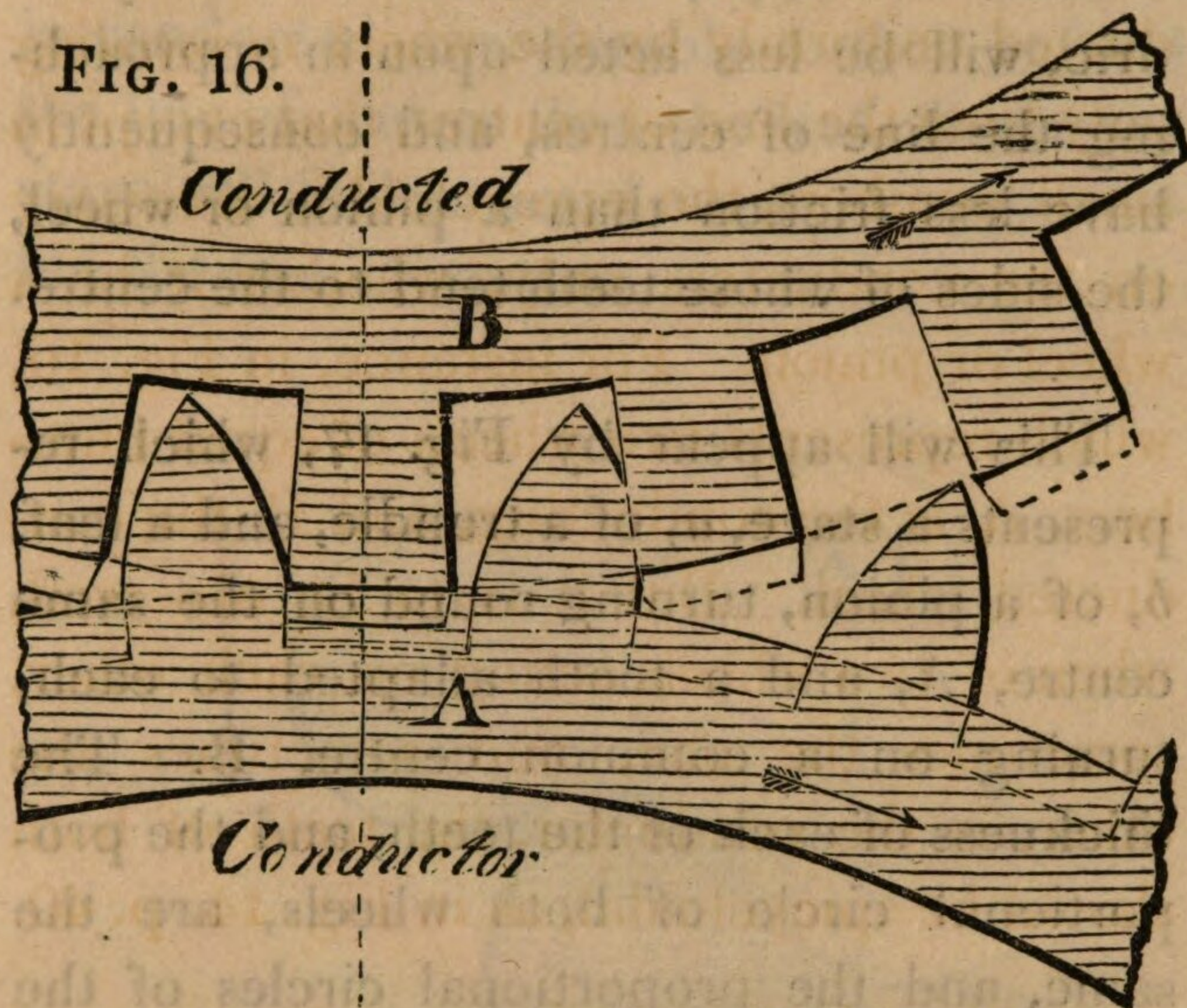
Whence it appears that a trundle with less than eight staves cannot be moved uniformly by a wheel with any number of teeth whatever.

The same is true also of stave-formed teeth (see Art. A. 32.) when the wheel drives the pinion. (Ed.)

29. From what I have already said, it will be evident, when the pinion consists

of such a number of teeth, as to be conducted uniformly by the wheel in *receding* only from the line of centres, that except in small numbers, the epicycloid is necessary on the conductors only, whether it be a wheel or pinion. For instance, in Fig. 16, which represents two wheels of equal numbers, A is the conducting, and B the conducted wheel. But it is to be observed, when of two wheels acting on each other, sometimes the one, and sometimes the other, is the conductor, the teeth of both should be epicycloidal, as in Fig. 14. p. 40.

When the teeth of the conducted wheel or pinion are acted upon by those of the conductor, in *receding* only, from the line of centres, it may be remarked, if they were perfectly made, and of durable materials, it would be unnecessary to extend the conducted teeth beyond their proportional circle. But these properties being unattainable, and as the angles which terminate their sides, would be apt to cut the conducting teeth, and occasion an irregu-



lar motion, it is proper to form the extremities of the teeth of the conductor, in the manner represented in the figure by the dotted lines.

30. Sometimes it may be requisite to have but few teeth in the pinion. In such cases, in the *conducted*, whether wheel or pinion, I would prefer staves to teeth, properly so called, or to leaves, because a

trundle or wheel, whose staves are cylindric, will be less acted upon in approaching the line of centres, and consequently have less friction than a pinion or wheel, the sides of whose teeth tend to the centre.

This will appear by Fig. 17, which represents a stave, *a*, of a trundle, and a leaf, *b*, of a pinion, turning round on the same centre, A, and a tooth adapted to each, turning on a common centre, B. The thickness of each of the teeth, and the proportional circle of both wheels, are the same, and the proportional circles of the pinions are also equal, and teeth are each made of the greatest length which the intersection of the curves will admit, which turns out considerably greater in the tooth adapted to the stave. The shaded parts represent the tooth adapted to, and acting upon, the stave; and the dotted lines represent the tooth adapted to, and acting upon, the leaf. The teeth, in both cases, are represented as just at the point where they would cease to move the leaves or

staves uniformly ; and it appears the stave is conducted considerably further beyond the line of centres than the leaf ; hence the stave will be less acted upon in approaching the line of centres.

FIG. 17.

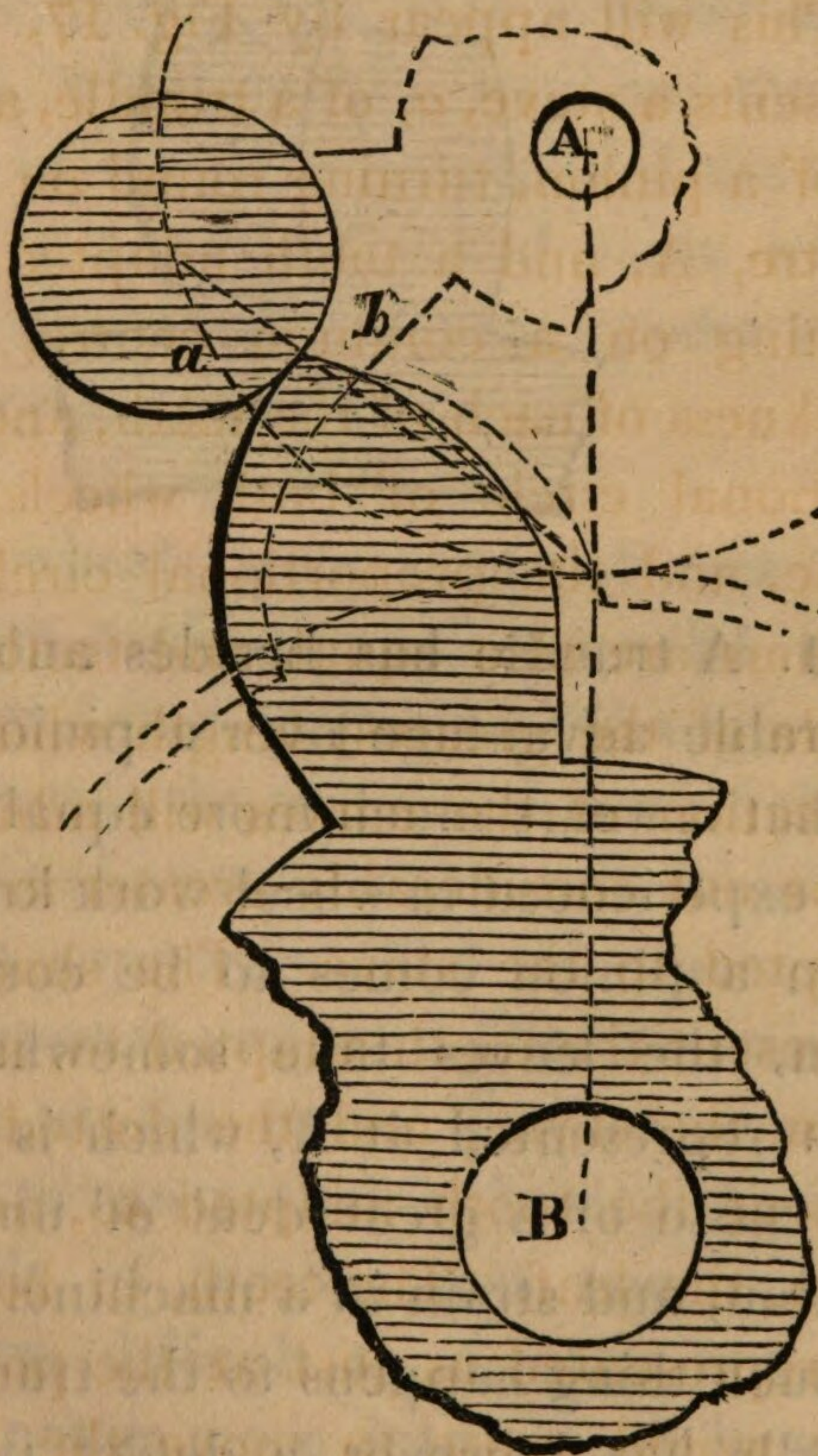
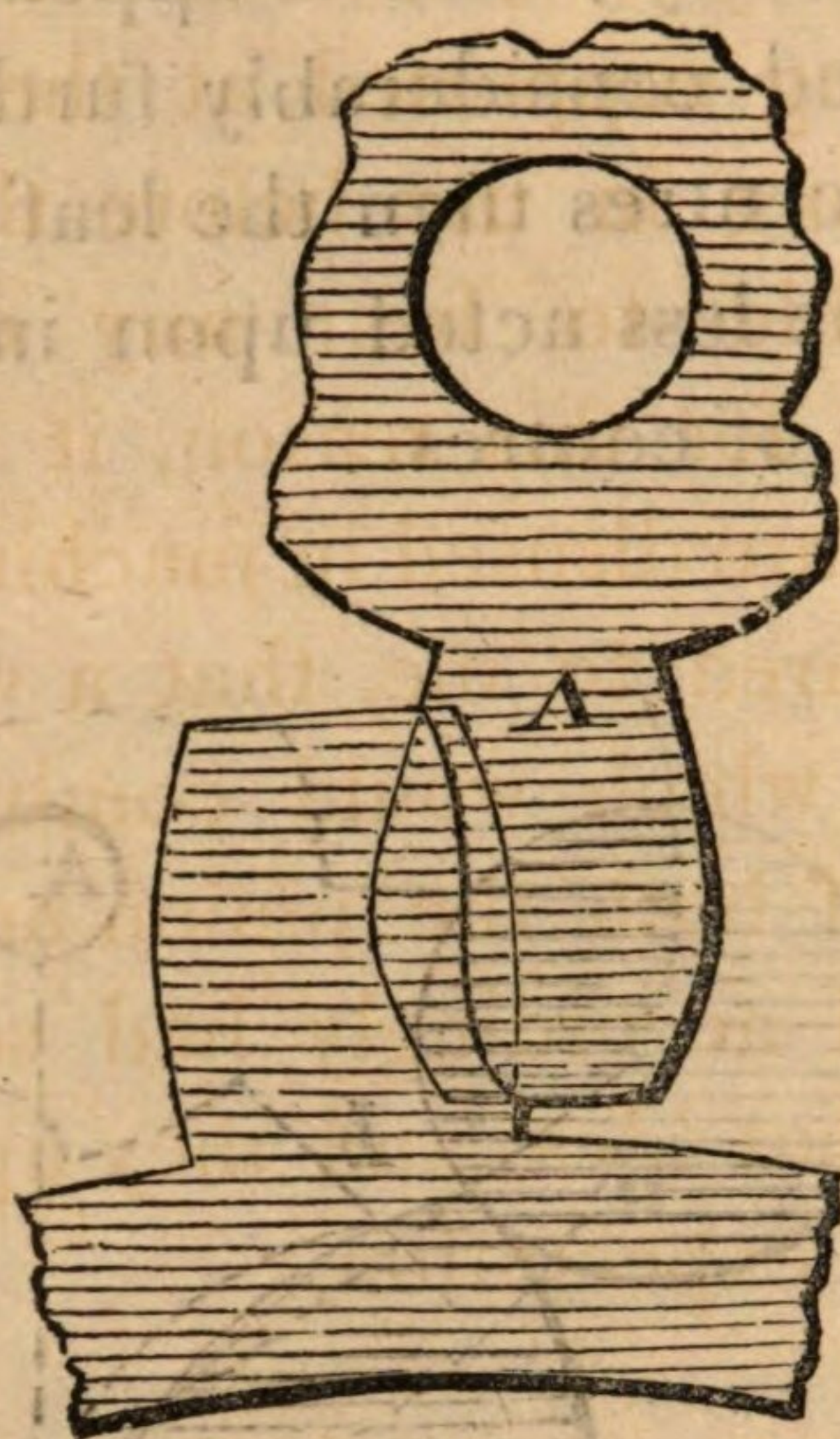


FIG. 18.



31. A trundle has besides another considerable advantage over a pinion ; which is, that it wears much more equally. Every one experienced in wheel work knows, that when a pinion comes to be considerably worn, the leaves take somewhat of the form represented at A, which is evidently the cause of a great deal of unnecessary friction, and strain in a machine. Whereas no such thing happens to the trundle. The trundle has, however, a defect perhaps as

bad, if not worse, than that of weakness. Its staves being supported at the ends only, are not long in use before they become quite unable to bear any considerable strain, and for this reason, it is now in a great measure disused in machines. It however appeared to me, that a wheel might be made, which would combine the advantages of both the pinion and the trundle, and I accordingly had some wheels made on that idea, and they appear to answer every expectation.

These wheels were made of cast iron. They were each cast of one solid mass. The upper figure represents the edge view, and the lower the section of one of them; whereby is shown the manner in which the teeth are supported, like the staves of a trundle at each end, and like the leaves of a pinion at the roots, but so very thin there, as to run no risk of having the common fault of pinions just now noticed. They were difficult to mould: but were they to come more into use, I have no

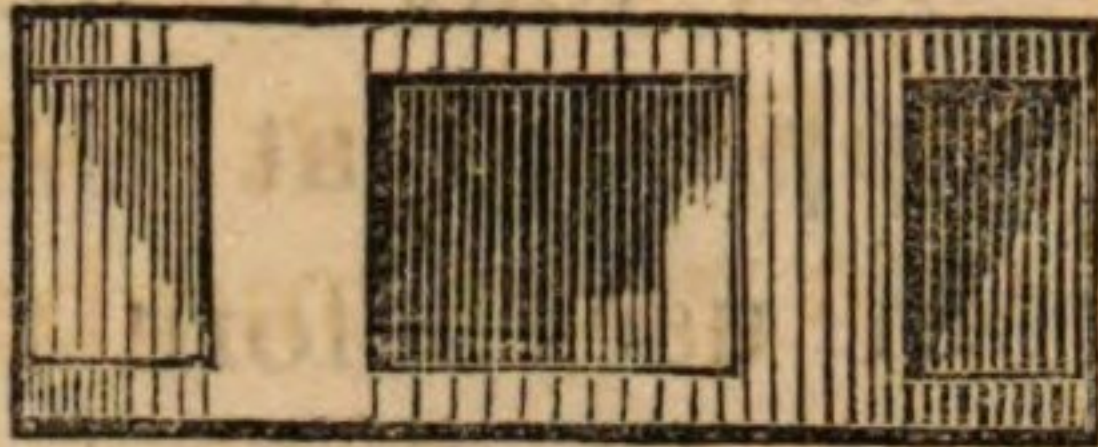
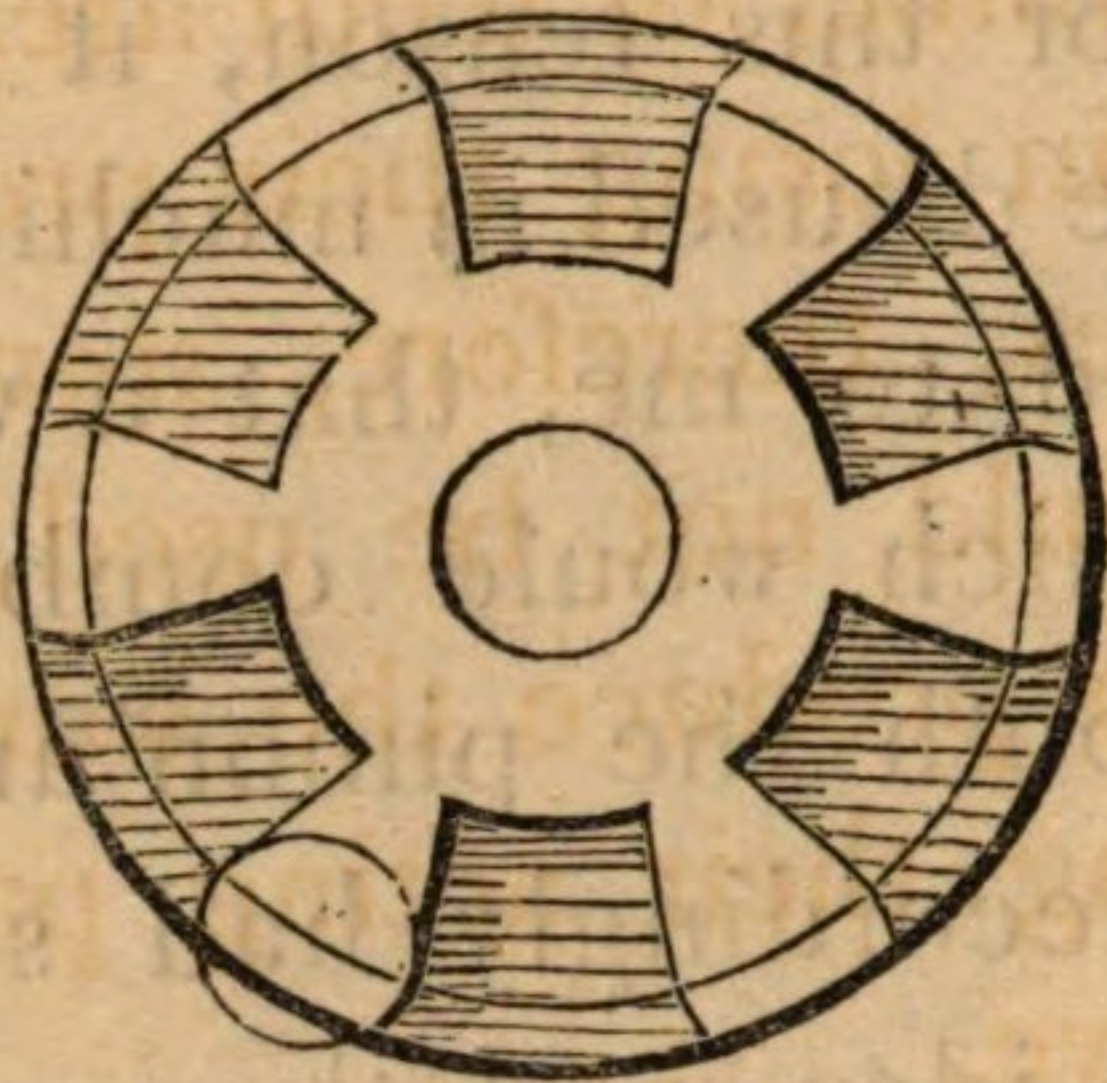


FIG. 19.



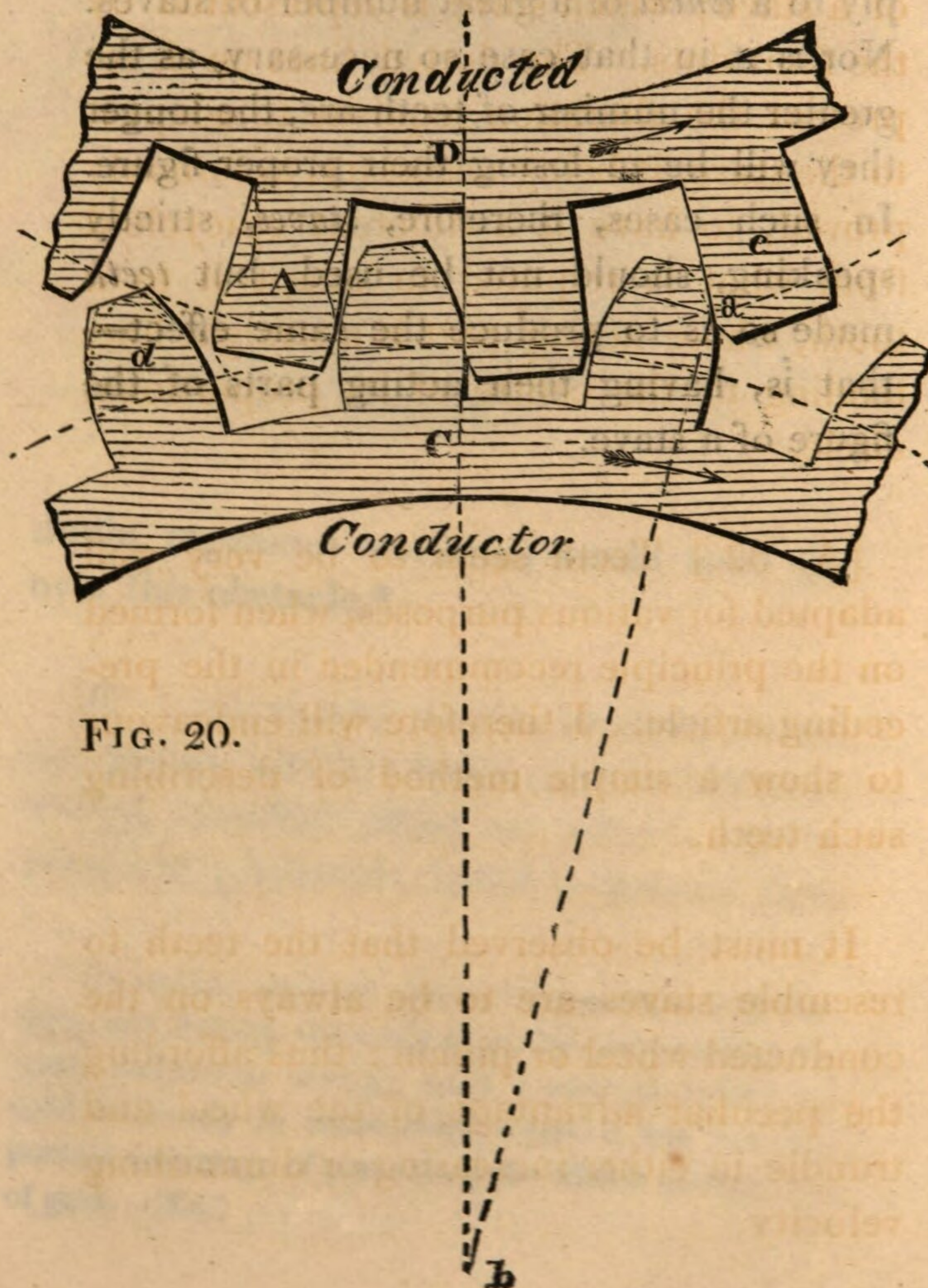
doubt ingenious workmen would soon get over this obstacle.*

32. I mentioned above, in cases where the pinion had few teeth, that in the *conducted*, whether *wheel* or pinion, staves should be preferred; but it is obvious, that

* By casting separate plates, with indents to fit the teeth, and bolting them together, the pinion might be made sufficiently strong: such a method indeed is used frequently in crane-work, where it has the important advantage of preventing the wheels getting out of geer. (ED.)

the method just described, of making a small trundle of cast iron, would not apply to a *wheel* of a great number of staves. Nor is it in that case so necessary, as the greater the number of teeth are, the longer they will be in losing their proper figure. In such cases, therefore, *staves*, strictly speaking, should not be used, but *teeth* made so as to produce the same effect—that is, having their acting parts of the figure of a stave.

What is meant will be better understood by inspecting the figure, where the lines

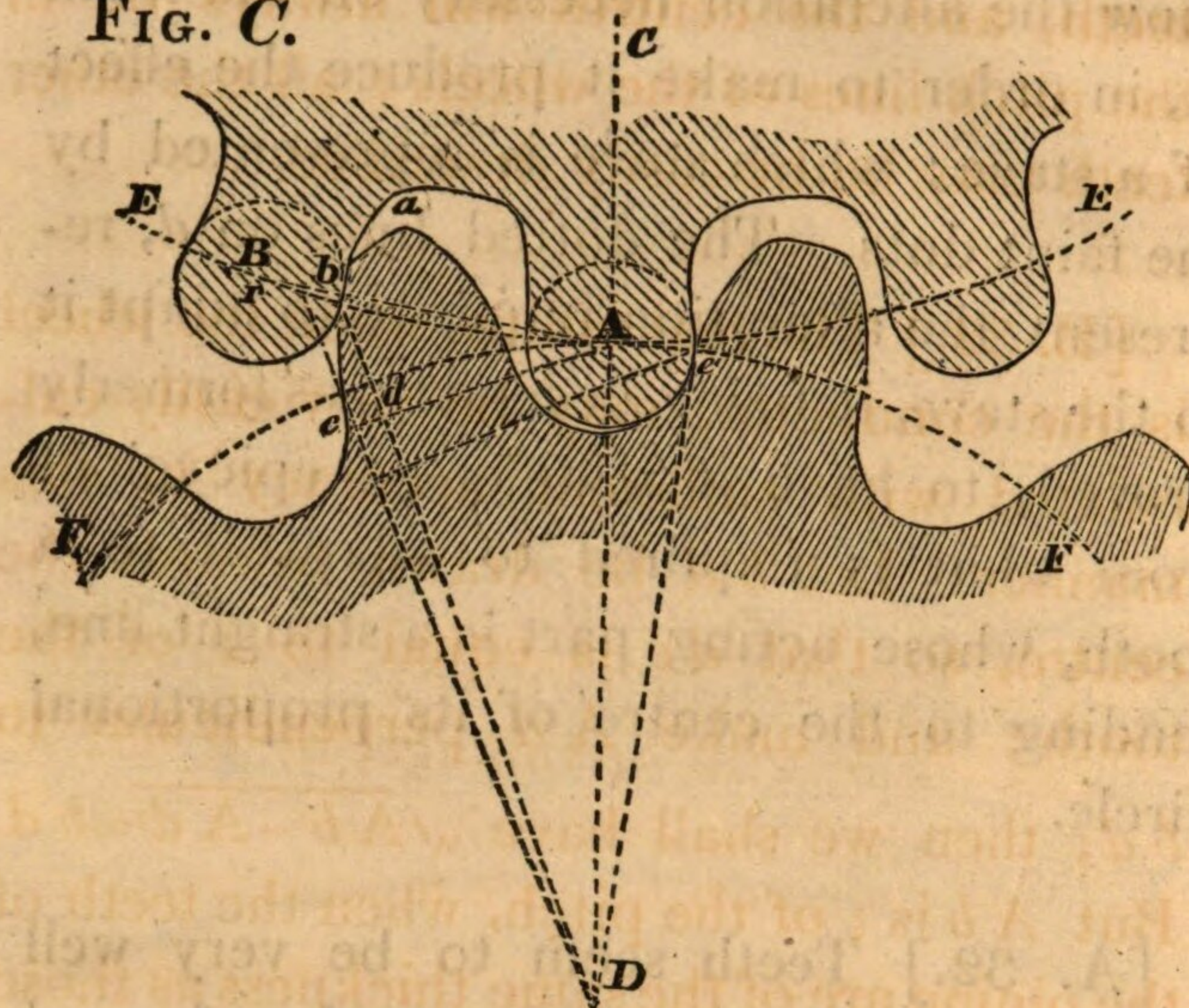


show the alteration necessary on the tooth A, in order to make it produce the effect of a stave; which stave is represented by the faint dots. The dotted lines on *d*, represent the alteration requisite to adapt it to the stave, it being necessary, as formerly proved, to have it a different epicycloid from what is required to adapt it to a tooth, whose acting part is a straight line, tending to the centre of its proportional circle.

[A. 32.] Teeth seem to be very well adapted for various purposes, when formed on the principle recommended in the preceding article. I therefore will endeavour to show a simple method of describing such teeth.

It must be observed that the teeth to resemble staves are to be always on the conducted wheel or pinion; thus affording the peculiar advantage of the wheel and trundle in either increasing or diminishing velocity.

FIG. C.



Let the teeth be divided as usual on the pitch lines, EE , FF ; and on the conducted wheel C describe circles, as though there were to be staves. Conceive the centre of one of these stave teeth to be in the line of centres at A , and draw the line AB joining the centres of the stave teeth. Then the radius Ab , from the centre A , will describe the curved side bc of the tooth of the conductor, and the curved part ba of the conducted wheel. And since this radius is equal to the pitch diminished by half the diameter of the circle of the stave

teeth, and the centres will always be in the pitch lines of the wheels ; all the other teeth may be easily described.

[B. 32.] The real radius, when the wheel is the conductor, may be very easily calculated with sufficient accuracy in this manner: Let bd be drawn towards the centre, so that dc is equal to $\frac{1}{4}$ of the tooth ; and make Ad perpendicular to bd ; then we shall have $\sqrt{Ab^2 - Ad^2} = bd$. But Ab is $\frac{1}{4}$ of the pitch, when the teeth of the pinion are of the same thickness as those of the wheel, and Ad is sensibly equal to $\frac{1}{16}$ of the pitch ; therefore if P = the pitch, $P \times \sqrt{\frac{9}{16} - \frac{49}{144}} = bd = .4714 P$. And as in practice we may always regard bd as the difference between the real and proportional radius ; it being only a very small quantity in excess,* we have this rule:

The real radius of a wheel, of the con-

* The excess is the difference between the radius and the secant of the angle containing $\frac{1}{16}$ of the pitch.

struction now described, should be equal to the proportional radius added to $\cdot 47$ times the pitch; and $\cdot 47$ times the pitch, is very little less than half the pitch. But when the teeth of the pinion are thicker than those of the wheel, the real radius may be less than is given by this rule.

The same rule applies to a wheel to drive a trundle.

I have chosen this approximate method in preference to a more accurate one, because the result is exhibited in those terms which are most directly comparable with the proportions founded on practical experience.

When the pinion is the conductor, the real radius should be the secant of the angle contained by the pitch, when the proportional radius is considered the radius of that angle.

Thus, ce is the angle contained by the

pitch, and eD is sensibly equal to the proportional radius; but by the form of the teeth re is perpendicular to eD , consequently rD is the secant to the arc or angle ce ; and the real radius required to impel the wheel till the succeeding tooth begins to act. (Ed.)

33. Hitherto the conducting teeth have been considered, as being always made as long as the epicycloidic form of their sides would admit. This however is not always necessary, and in some cases may be improper.

It now remains to show, the smallest real radius a wheel adapted to a trundle can have, without destroying the uniformity of the motion.

When the stave E (Fig. 13, p. 35.) shall have been conducted to the situation in which it is represented, the point E of the following stave, shall be in the line of centres GF . The stave A , in its turn, may

then be conducted by the following tooth, T Y V, and then it shall no longer be absolutely necessary, that the tooth R O S conduct the stave E. The tooth R O S may therefore be terminated in the point X, where it shall touch the stave E, when the point T of the following stave shall be in the line of centres, and the distance X F of this touching point, from the centre of the wheel, shall be the least real radius which can be given to the wheel.

To determine the point X, draw from the centre of the stave E to the point T, the straight line E T, and where this line meets the circumference of the stave E, you have the point required. *

34. The smallest real radius which can be given to a wheel, adapted to the leaf of a pinion, or the teeth of a wheel, must evidently be terminated by that point *a* of

* See Art. A 25 and B 32, where such proportions as are applicable in practice are given. (Ed.)

its tooth, which is in contact with the tooth or leaf *e*, after it has conducted it just until the tooth following begins to act: thus *a b* is the smallest real radius of the wheel C. *

But, in practice, perfect accuracy is not to be expected; and though it were even practicable to have wheels perfectly accurate when new, yet the moment they are put in motion they begin to wear, and deviate from the true figure of their teeth. It would therefore be attended with bad consequences, to make the real radius no greater than what we have here determined it to be, which is the least which

* When the wheel is conductor, if we pursue the same mode of calculation as in Art. B 32, making $P =$

the pitch, we shall have $P \times \sqrt{1 - \frac{25}{36}} = .553 P$. That

is, when the teeth of the conductor do not begin to act till they arrive at the line of centres, the real radius should be equal to .553 times the pitch added to the proportional radius. But when the pinion is conductor, the proportional radius is to the real radius, as the radius to the secant of the arc equal to the pitch. (Ed.)

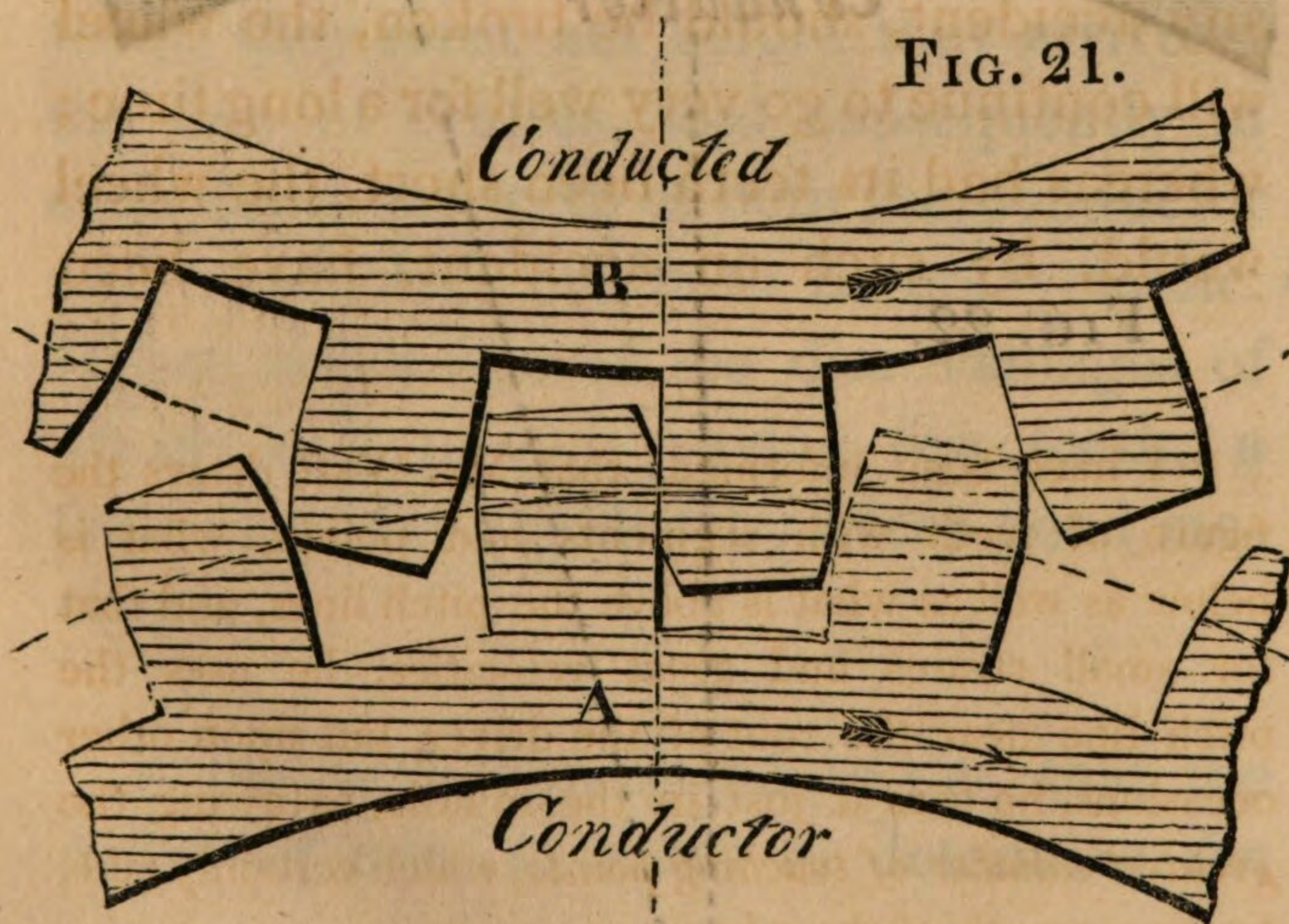
can be given. How much greater it should be, may be determined by circumstances.*

35. But it appears to me, that when wheels are made with their conducting teeth only epicycloidic, and the motion is steady, there is not much danger of their being too long; for the longer the teeth are, the greater number of them will be in action at the same time, and consequently the strain will be more general, which will cause them to retain their true form longer. Besides, though a tooth, from any accident, should be broken, the wheel will continue to go very well for a long time; whereas had its teeth been short, the wheel would, by such an accident, have been

* I have been informed, that Mr. Watt draws the figure of teeth with segments and points, what is below as well as what is above the pitch lines, and that for small strains and great velocities, he uses the pitch line near the root of the driver, but upon other occasions, he uses it just in the middle, as giving the *greatest number of touching points*, which certainly adds to the strength of the wheels.

rendered useless. I am however aware, that very long teeth are less able to sustain any sudden stress upon their extremities.*

But even supposing the teeth, made in the manner above described, to be no longer than those formed epicycloidic, upon both the conductor and conducted, yet the former will have less friction, and consequently wear longer than the latter, and if they be of the same length, and the same thickness at the roots, they must be equally strong. Let Fig. 21 repre-



* See the last paragraph of Art. 51. (Ed.)

sent wheels having the teeth of both conductor and conducted epicycloidic. In Fig. 22, those of the conductor only are

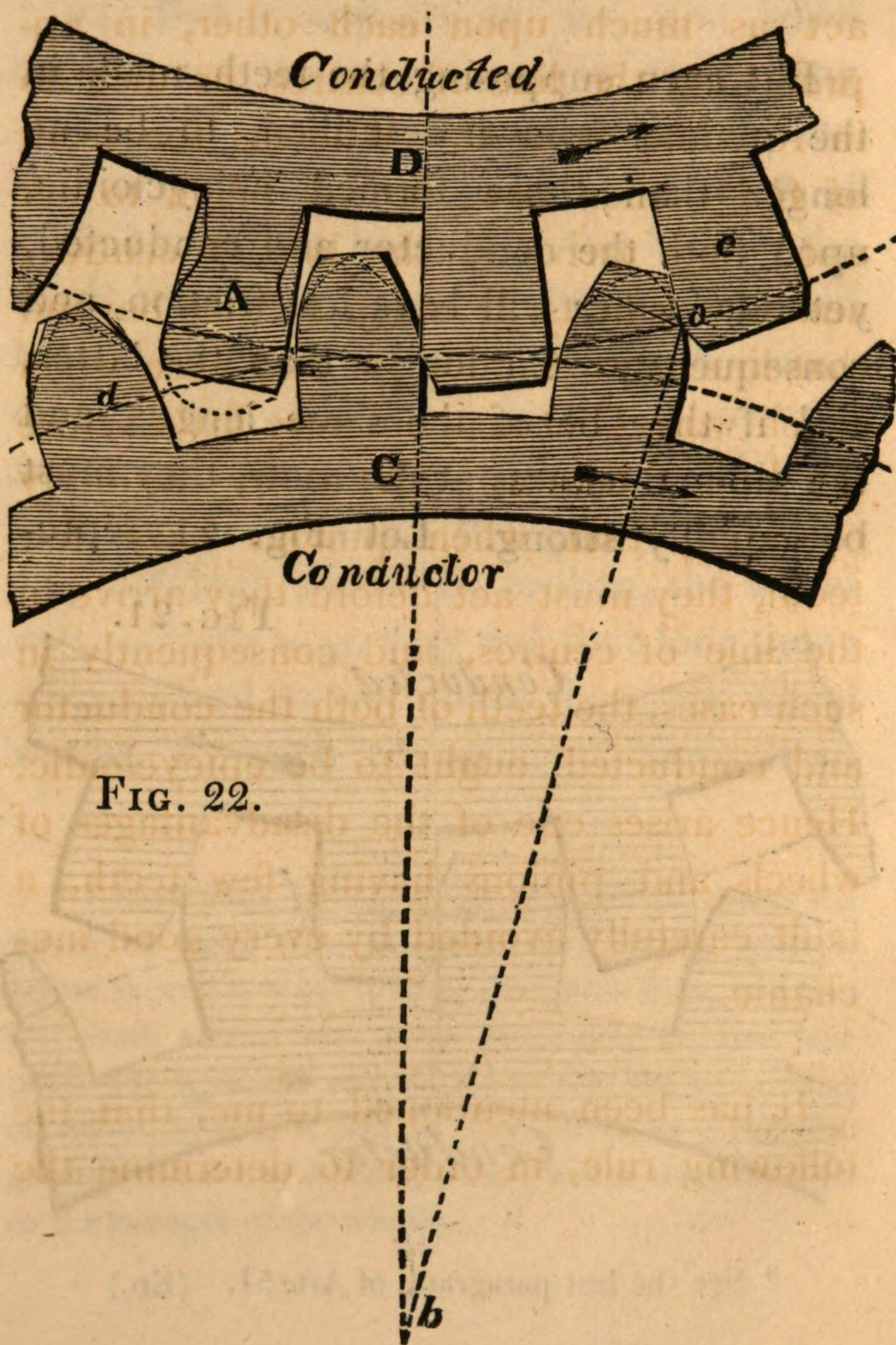


FIG. 22.

epicycloids, and by inspecting the figure, it will be made obvious, that the teeth of A B, even when the wheels are new, must act as much upon each other, in approaching the line of centres, as they do in receding from it. Whereas the teeth of C D, when new, do not begin to act until they arrive in the line of centres; and C conducts D much further beyond that line than A does B: and even when much worn C D acts but very little before the line of centres. But it was formerly observed, that when pinions have few teeth, they must act before they arrive in the line of centres, and consequently in such cases, the teeth of both the conductor and conducted, ought to be epicycloidic. Hence arises one of the disadvantages of wheels and pinions having few teeth, a fault carefully avoided by every good mechanic.

It has been mentioned to me, that the following rule, in order to determine the

length of the teeth of wheels, is employed by the ingenious Mr. Murray of *Leeds*. *

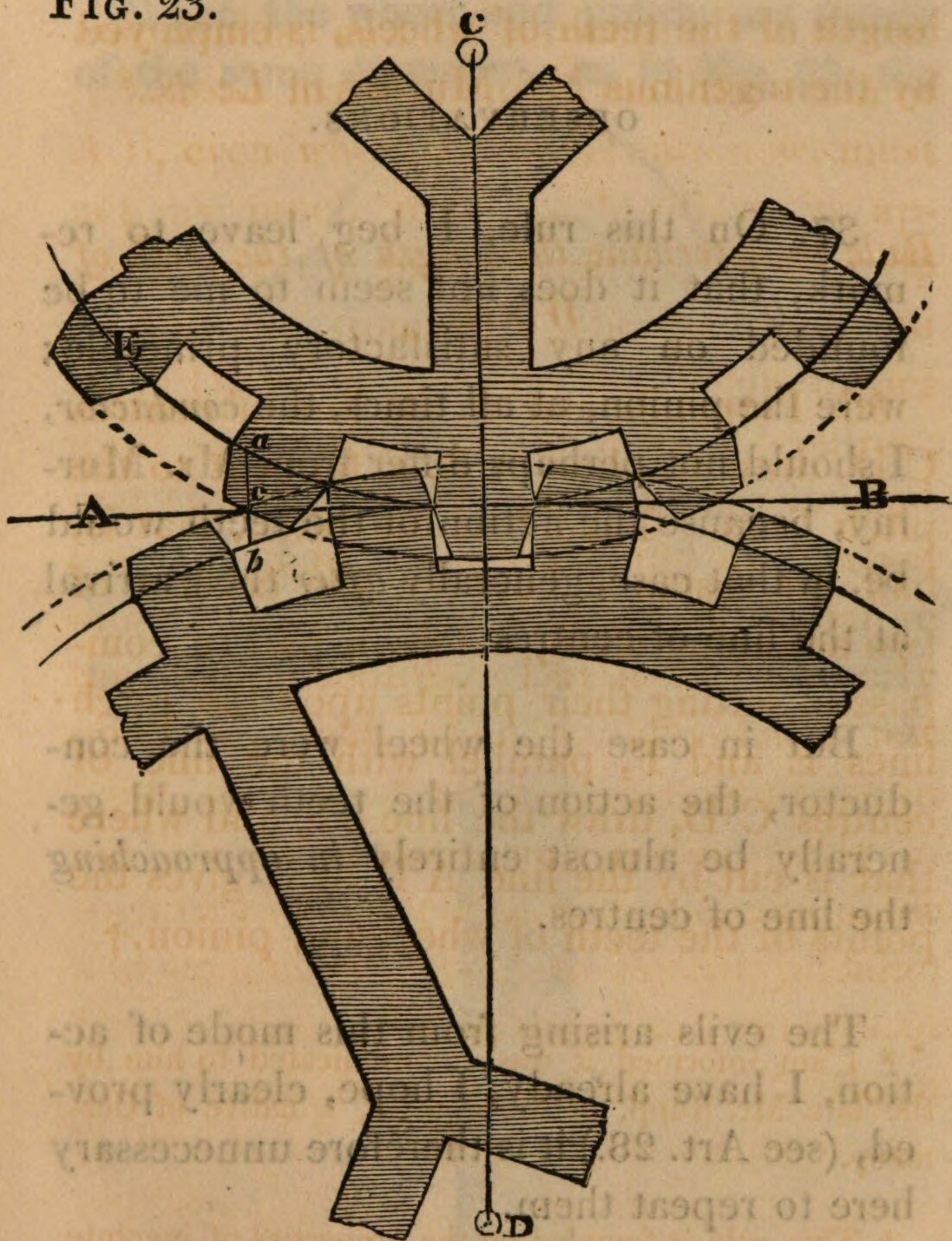
*Rule to determine the length of the teeth of
Wheels.*

36. Perpendicular to the line of centres C D, draw the line A B, a tangent to the pitch lines. Take half the pitch, that is, half the distance between the centres of two adjoining teeth, within a pair of compasses, setting their points upon the pitch lines E and F, parallel with the line of centres C D, draw the line *a b*, and where that is cut by the line A B at *c*, gives the points of the teeth of wheel and pinion. †

* I am informed it was communicated to him by the late Mr. Rupp, of Manchester, a native of Germany.

† This rule is founded on the properties of involute teeth; for it may easily be proved that the line *a b* will always be divided in the same ratio as the pitch line divides the distance between the centres of the wheels;

FIG. 23.



and that the pitch line always divides the length of the teeth in that ratio, in involute teeth. Hence the observations in the following article of the text, are to be

OBSERVATIONS.

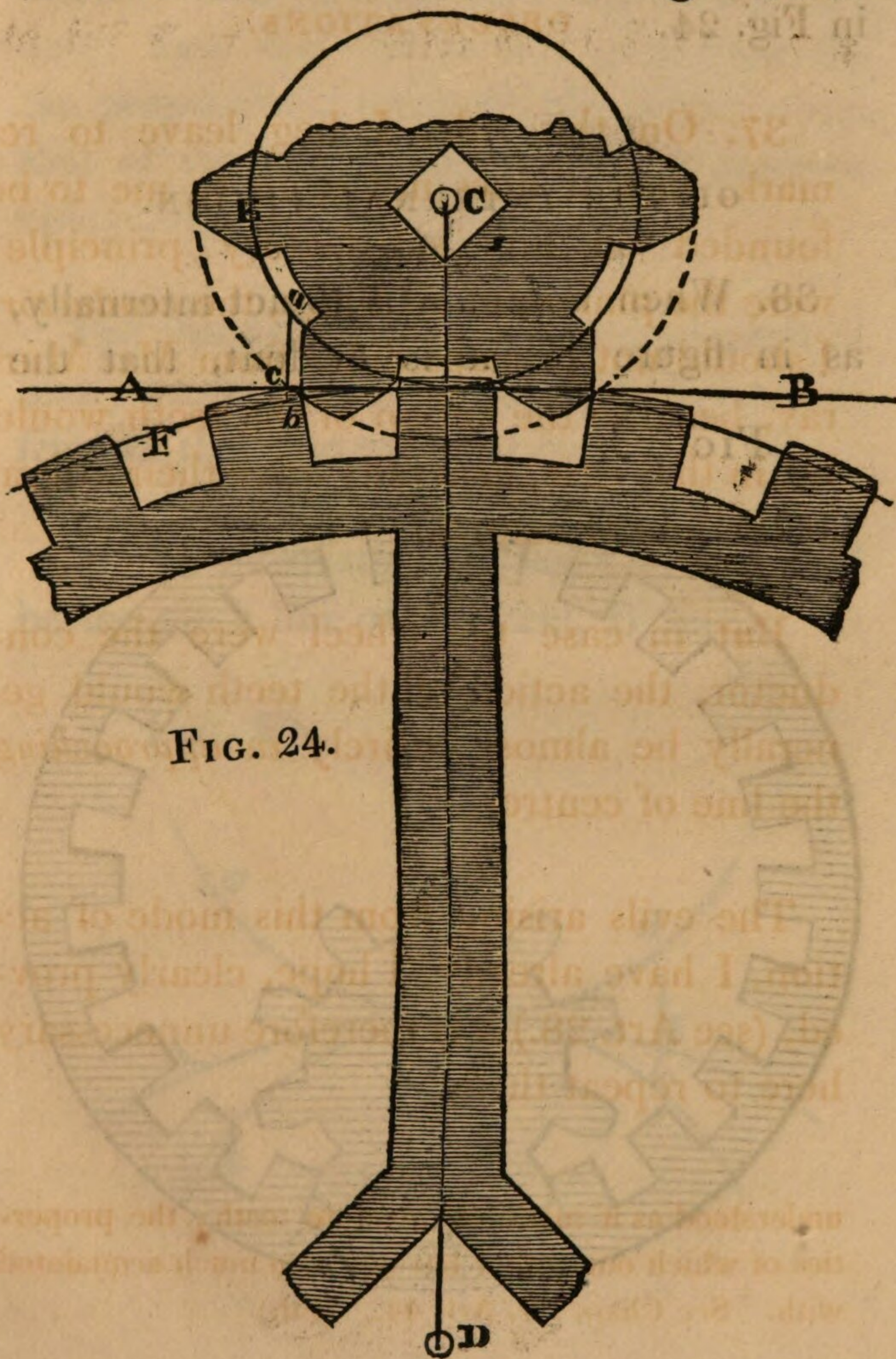
37. On this rule, I beg leave to remark, that it does not seem to me to be founded on any satisfactory principle; were the pinion, at all times, the *conductor*, I should not perhaps differ from Mr. Murray, because the action of the teeth would be, in that case, generally *after* their arrival at the line of centres.

But in case the wheel were the conductor, the action of the teeth would generally be almost entirely *in approaching* the line of centres.

The evils arising from this mode of action, I have already, I hope, clearly proved, (see Art. 28.) it is therefore unnecessary here to repeat them.

understood as if made on involute teeth; the properties of which our author has not been much acquainted with. See Chap. III. Art. 44. (Ed.)

When the wheel and pinion are nearly of the same diameters, as in Fig. 23, the

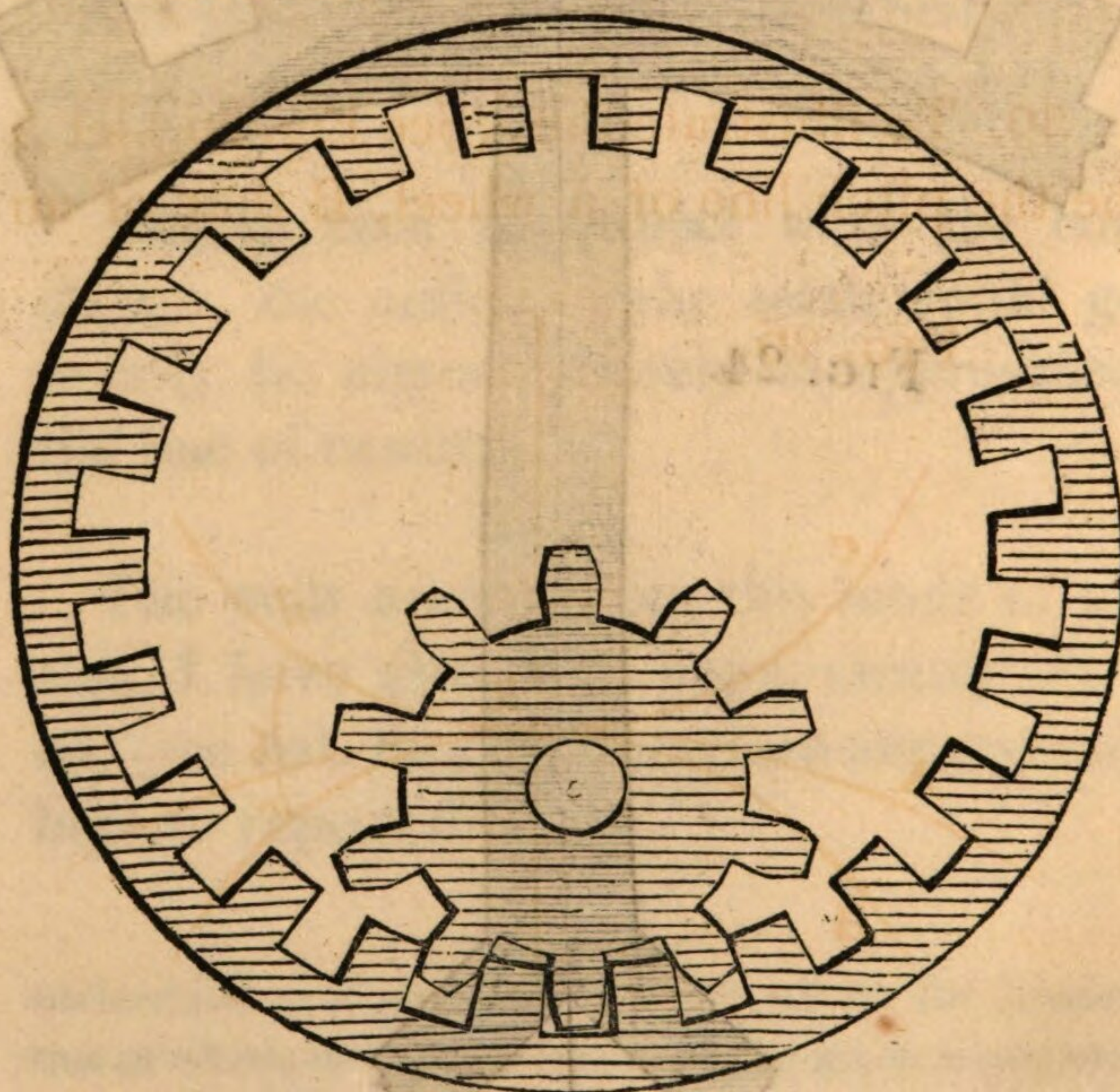


effects are not so obvious as when the pinion is much smaller than the wheel, as in Fig. 24.

OF THE INTERNAL PINION.

38. When a pinion is to act internally, as in figure 25, it is evident, that the

FIG. 25.

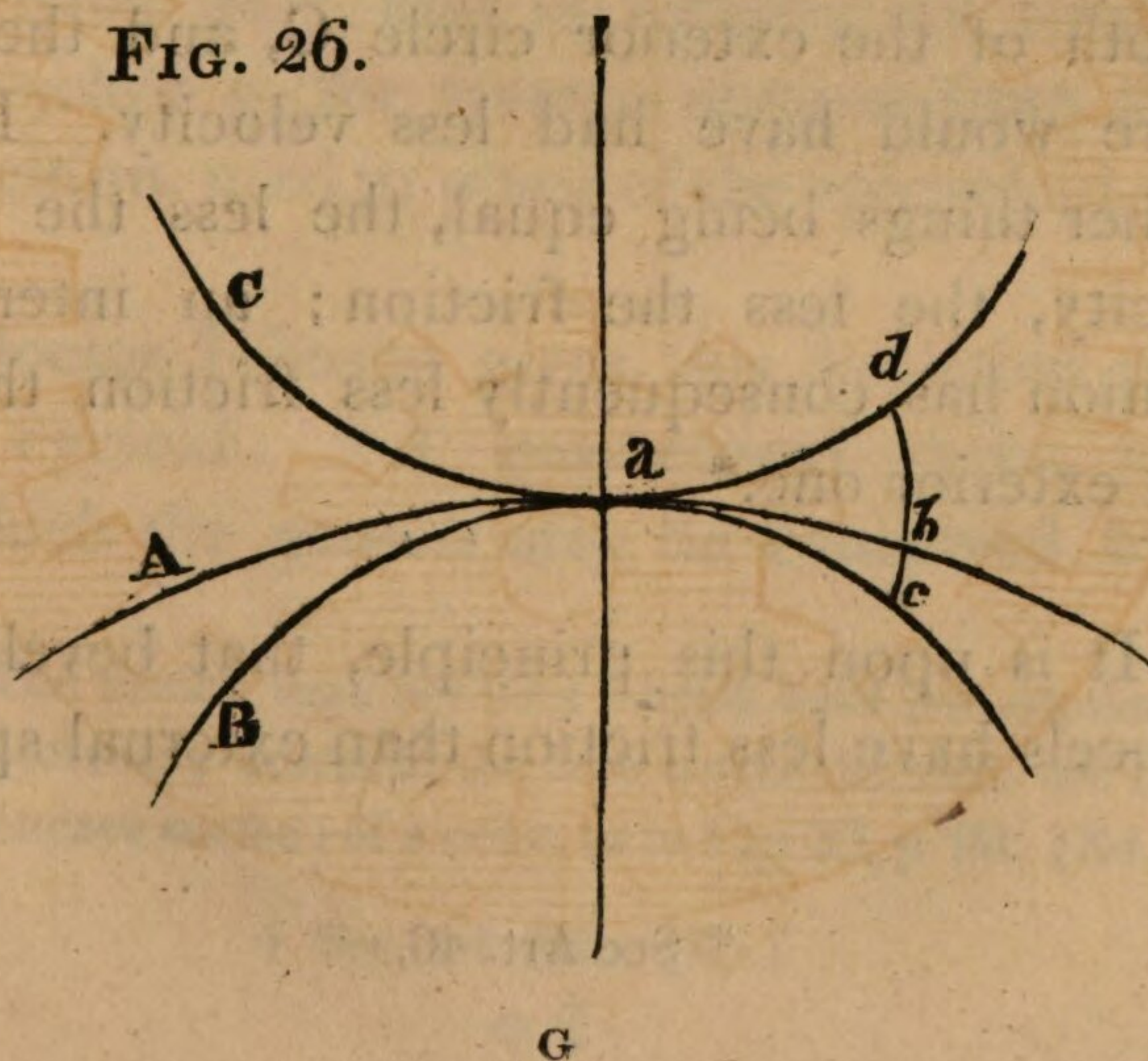


teeth may be formed on the principles already laid down, with this difference only, that the epicycloid generated by the proportional circle of the pinion upon that of the wheel, should be an interior epicycloid.

The internal pinion may be adopted in many cases with advantage, as it has less friction than the external one.

39. To illustrate this, (See Fig. 26,) let A be the pitch line of a wheel, B that of an

FIG. 26.



internal pinion, and C that of an external pinion.

Suppose the circle A to be moved till the point *a* arrives at *b*, and that the points *c d*, in the circles B C, have both moved over a space equal to *a b*. Now it is evident, that the distance from *c* to *b* is much less than that from *b* to *d*, and consequently had the circles moved one another by means of teeth, a tooth of the interior circle B, in the same part of a revolution, would have slid over a smaller part of a tooth of the circle A, than a tooth of the exterior circle C, and therefore would have had less velocity. But other things being equal, the less the velocity, the less the friction; an interior pinion has consequently less friction than an exterior one.*

It is upon this principle, that bevelled wheels have less friction than external spur

* See Art. 46.

wheels ; bevelled wheels acting in a mean situation between external and internal spur wheels.*

Of the Rack and Pinion.

40. What is called the rack and pinion, is used for various purposes in mechanics ; as in jacks for raising great weights, and for the opening and shutting of sluices.

The rack and pinion should be made upon the principles of spur geers ; with this difference only, that in forming the teeth, the *cycloid* is, for reasons obvious from its definition, used in place of the *epicycloid*.†

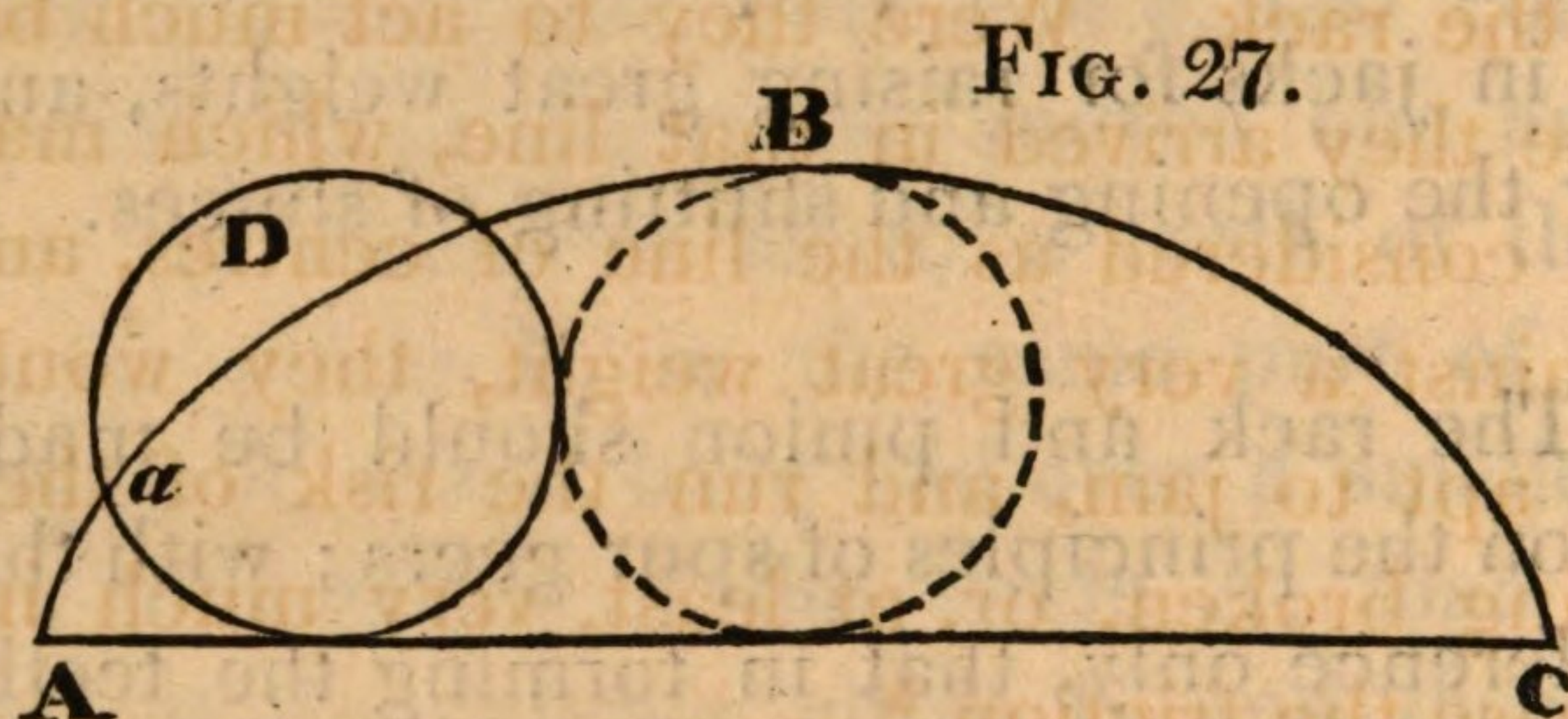
Doctor Johnson gives this definition of the *cycloid* : “ A geometrical curve, of
“ which the genesis may be conceived by

* The notion that bevelled wheels have less friction is not correct ; unless in the case where the teeth are in the concave surface of a cone, as in Fig. 32, p. 90. (Ed.)

† See Art. 52. (Ed.)

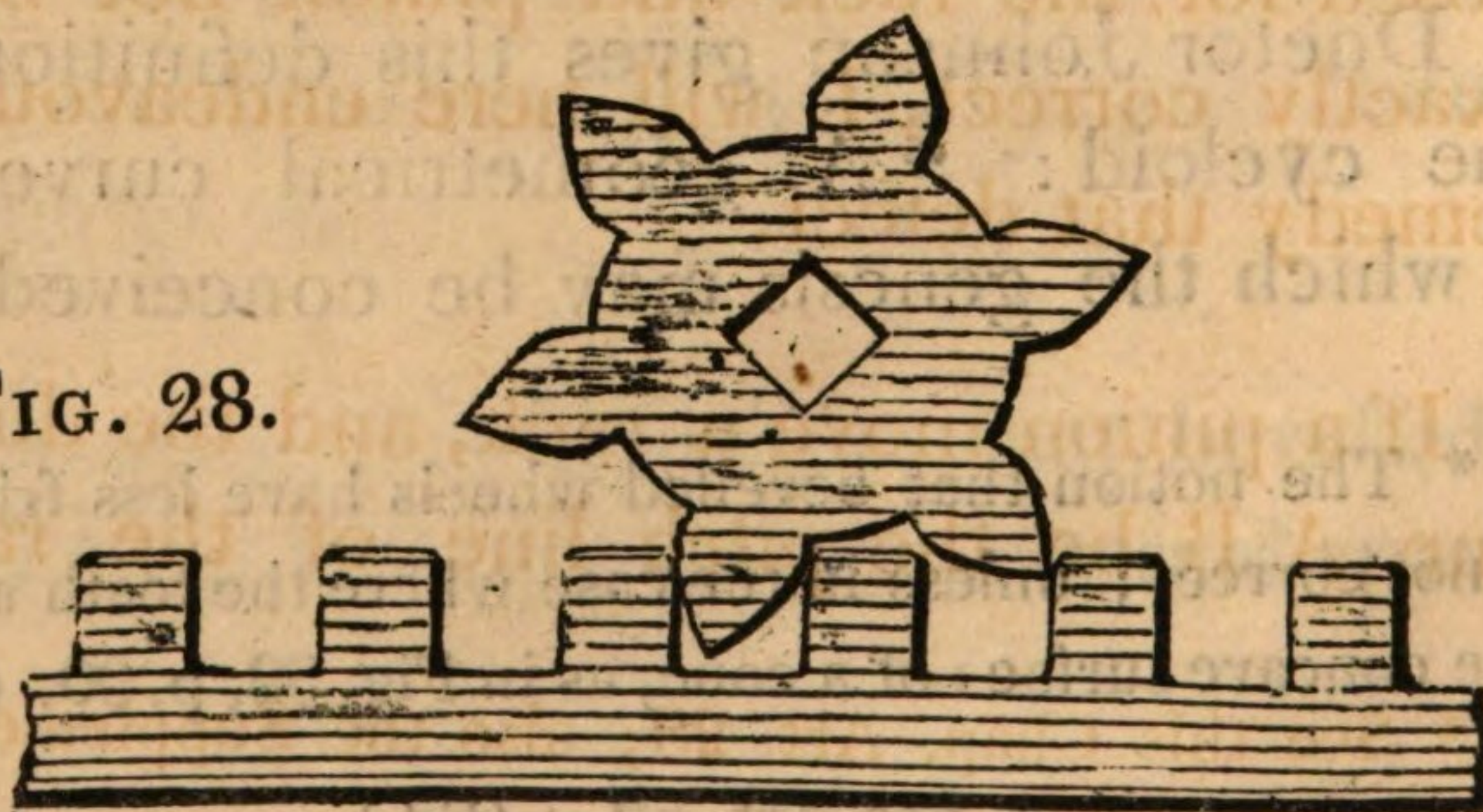
“imagining a nail in the circumference
 “of a wheel: the line which the nail de-
 “scribes in the air, while the wheel re-
 “volves in a right line, is the cycloid.”

Thus A B C, is a cycloid generated by
 the point *a*, in the circle D, while it revolves
 on the right line A C.



The figure represents the teeth of a rack
 and pinion, formed in what seems the best

FIG. 28.



mode in cases where a great weight is attached to the rack.

The leaves of the pinion are made as long as the curve will admit, in order to prevent them from beginning to act before they arrive in the line passing through the centre of the pinion, perpendicular to the rack. Were they to act much before they arrived in that line, which may be considered as the line of centres, and against a very great weight, they would be apt to jam, and run the risk of their being broken, or, at least, very much increase the friction.*

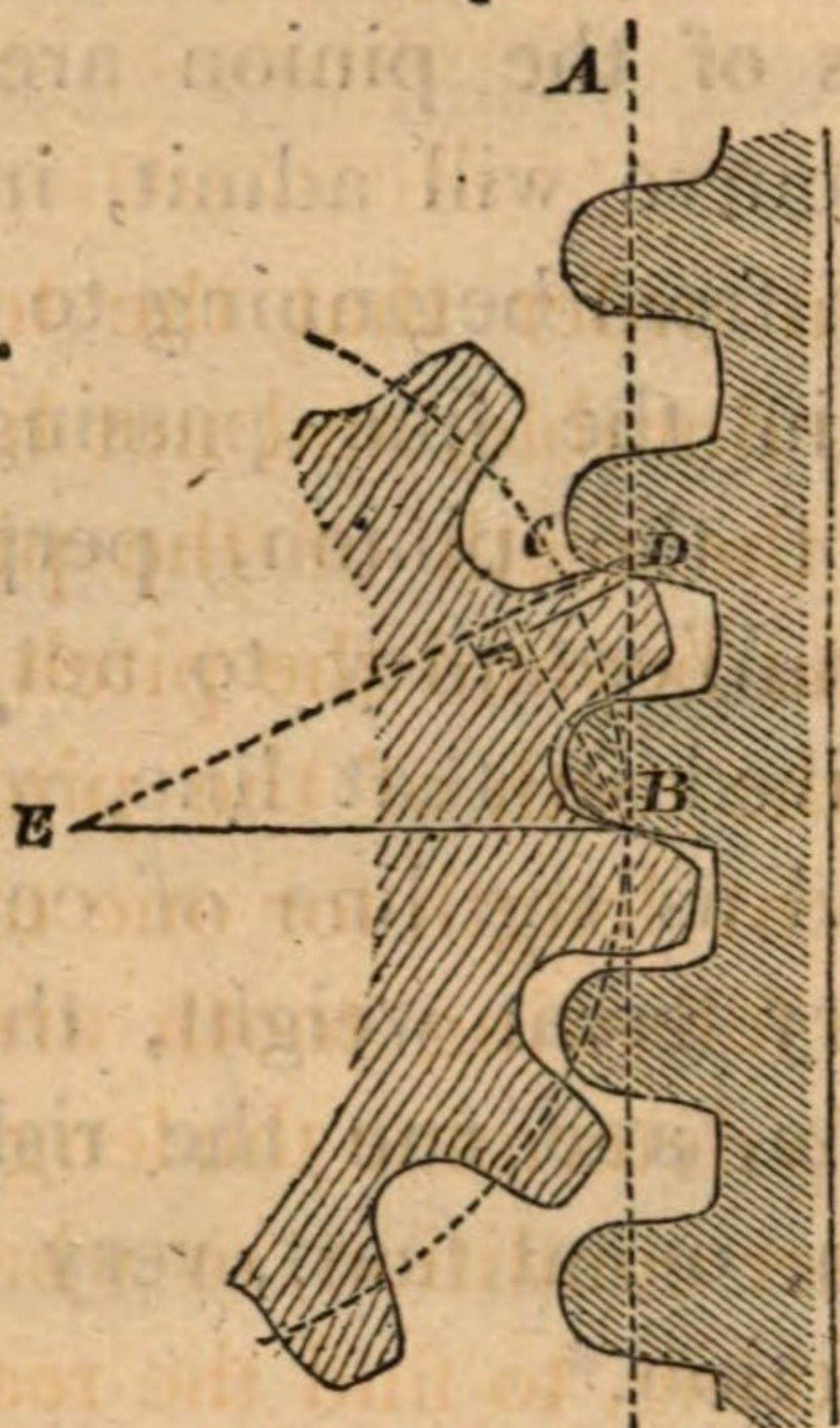
[A. 40.] The construction above proposed for the rack and pinion not being exactly correct, I will here endeavour to remedy that defect.

If a pinion move a rack, and the dotted line A B be the pitch line of the rack;

* See this Chap. Art. 28; also the Supplementary Observations.

and the dotted circle CB the pitch line of the pinion; then the curved side CD of the tooth of the pinion should be an

FIG. D.



involute of a circle. Or, it is that curve which a point in a cord would describe on the pinion, were the pinion turned by drawing the cord constantly in the direction AB ; the cord being supposed to be wound round the pitch circle of the pinion. Now as BD is the part of the cord which unwinds from the arc CB , it is obvious that the pitch lines move with equal velocities; and the force acting con-

stantly at the same distance from the centre of motion, the force will be constant, except that variable part which is lost in friction.

[B. 40.] In order that the teeth of the rack may be durable, and not liable to cut the face of the teeth of the pinion, they should extend beyond the pitch line of the rack ; and the teeth of the pinion should be of sufficient length for one to move the rack, till the following tooth arrives at the point B. To determine the length that will fulfil the latter condition, or what amounts to the same thing, to find the real radius of the pinion, we may suppose a line drawn from the point C, to the centre E of the pinion ; also make F B perpendicular to C E, and F D perpendicular to F B. Then, by similar triangles we have F B : B D :: B E : the real radius = $\frac{BD \times BE}{FB}$. But B D is equal to the pitch, and F B to five sixths of the pitch, therefore the real radius = $\frac{6 \times BE}{5}$. That is, the real radius should be six fifths of the proportional radius.

In ordinary cases, the curved surfaces of the teeth of the pinion may be described from centres in the pitch circle, with the radius DB . And instead of making the teeth of the rack square to the pitch line, they may be described by the same radius BD , from points in the pitch line of the rack; and the ends of the teeth as well as the hollows to receive them, in the pinion may be semicircular. This mode of forming the teeth will make them very strong without affecting the motion.

[C. 40.] When the rack impels the pinion, the curved face of each of the teeth of the rack, should be a portion of a cycloid (as *Aa* Fig. 27.) and the leaves of the pinion straight lines radiating from the centre of the pinion; the diameter of the generating circle for describing the cycloidal teeth should be half the proportional diameter of the pinion. (Ed.)

SECTION II.

Of Bevel Geer.

41. Hitherto our inquiry has been confined to what is called spur geer, or the action of wheels and pinions whose axes are parallel: we come now to speak of what is called *bevel geer*, or the action of wheels of which the axes are angular to each other. As we formerly regarded the action of spur geer, with teeth indefinitely small, as the rolling of cylinders upon the surface of each other, we may now regard the action of bevel geer with such teeth, as the rolling of cones in a similar manner.

In order to illustrate this, let us suppose it is required to make one wheel move another, the axes of which are not parallel.

FIG. 29.

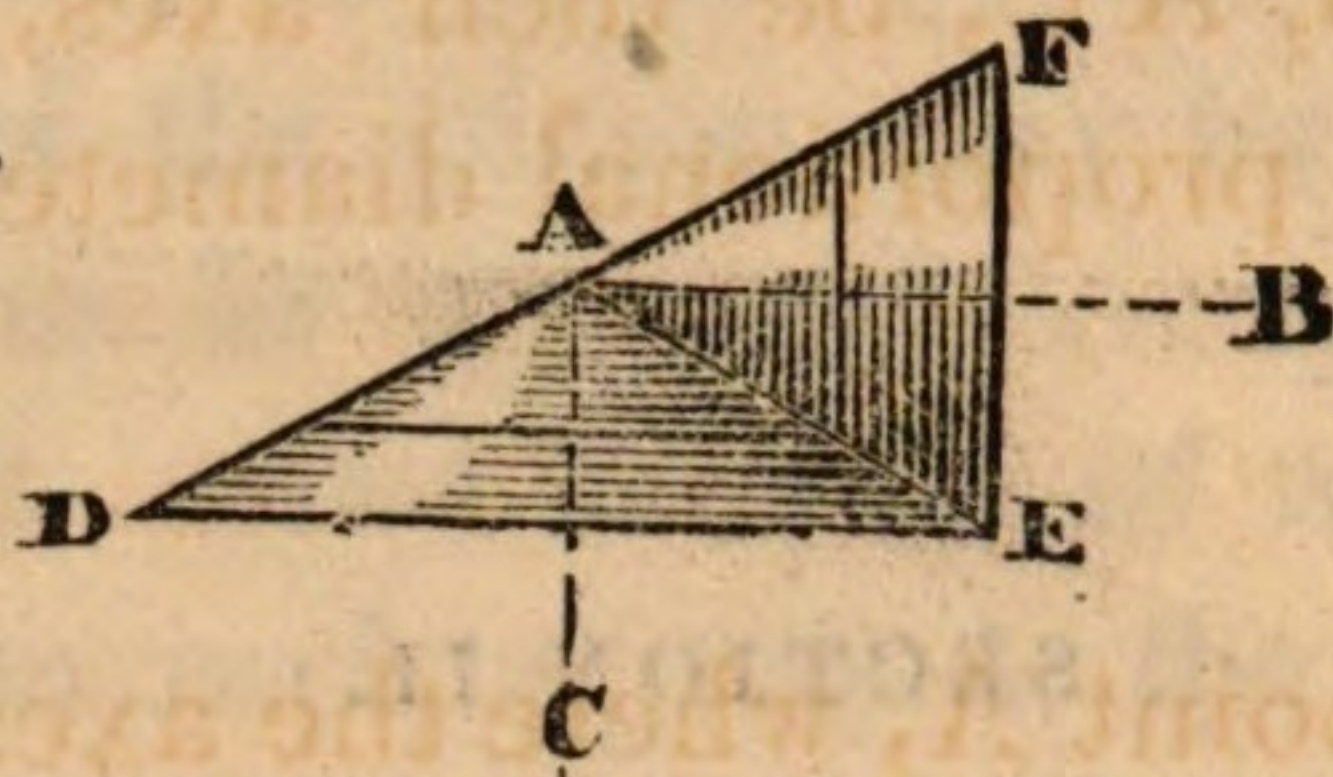


FIG. 30.

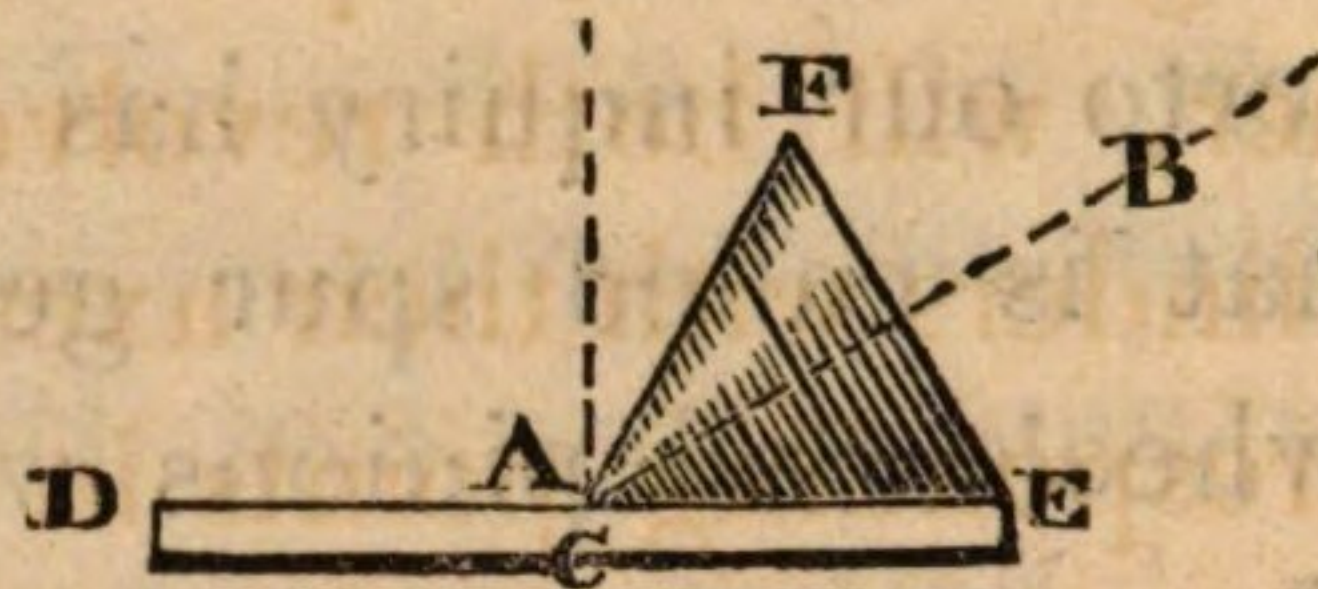


FIG. 31.

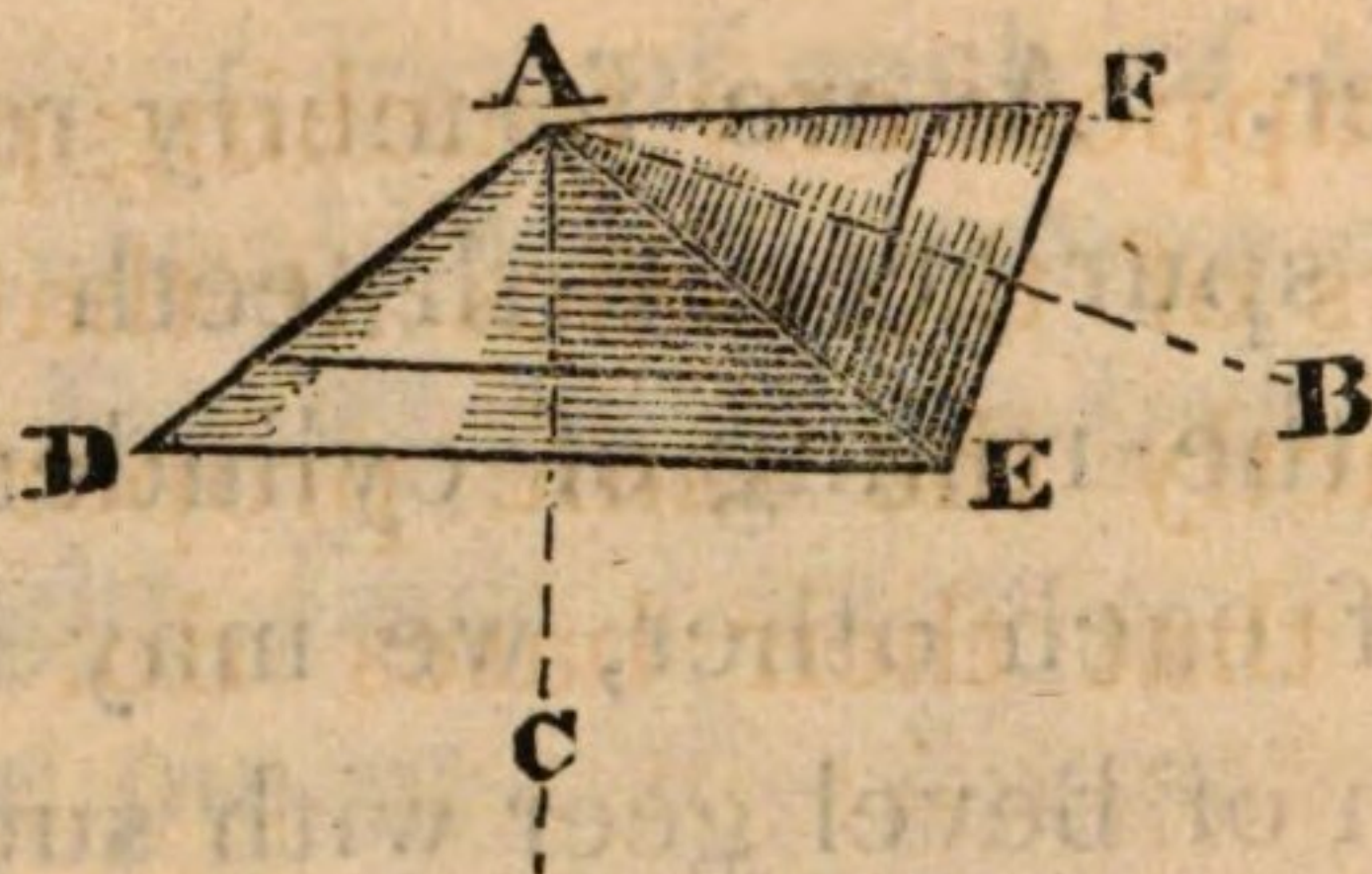
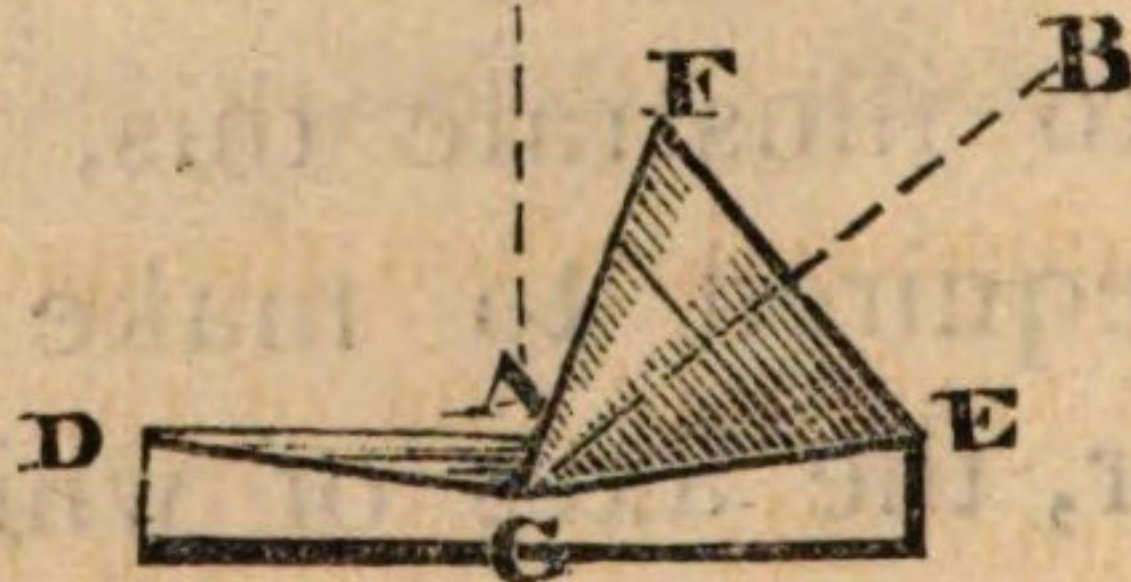


FIG. 32.



Let $A B$, $A C$, be their axes, and $D E$, $E F$, their proportional diameters or pitch lines.

To the point A , where the axes intersect, draw $A E$, $A F$, $A D$; then $D A E$, and $E A F$, shall be the outline of two cones,* which rolling the one upon the surface of the other, will both revolve, so that, like two cylinders with their axes parallel,† all the corresponding points in each, shall move in every part of their revolution with equal velocity.

For, suppose any touching point D , the diameters of the cones at that point shall bear exactly the same proportion to one another, that their bases do. The same

* These cones we shall call the proportional cones of the wheel and pinion.

† It is to be observed, when the axes are in certain inclinations, the surface of one of the cones becomes concave, as in figure 32; in others, as in figure 30, the point A is in the same plane with $D E$.

may be said of every other point on their surfaces, and consequently they shall revolve in the same manner as two cylinders having their axes parallel, and the cones may be considered as bevel wheels with indefinitely small teeth.

But in practice, we require finite and sensible teeth: in bevel geer, these are made similar to those of spur geer, with this difference, that in spur geer, they are parallel; but in bevel they must, as is evident, diminish in length and thickness, as they approach the summit of the cone.

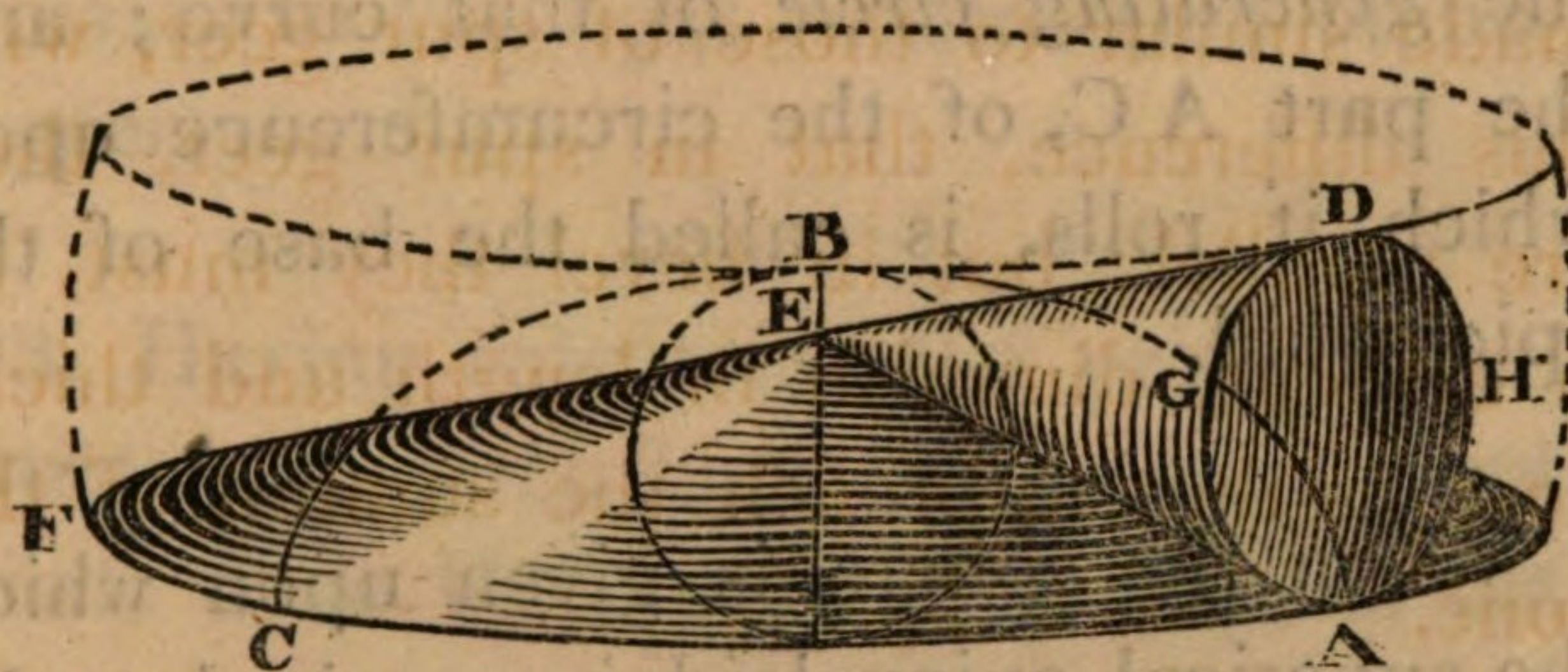
The teeth may be made of any breadth, according to the strength required, and they are thereby enabled to overcome a much greater resistance, and work smoother, than is possible for a common face wheel and trundle, which, for that reason, are now superseded by bevel geer.

42. The epicycloid, which gives the

true curve to the teeth of bevel geer, differs from that used in spur geer, in being generated by the rolling of one cone upon the surface of another, while their summits coincide.

Thus for example, in the figure, the

FIG. 33.



curve, A B C, is described by a supposed style, fixed in the point A, of the circumference of the base of the cone A D E, while it rolls upon the cone F C A E.

The style A, being always at the same distance from the point E, where the summit of the cone is fixed, all the points of the curve A B C, shall be equidistant from

the point E, and consequently upon the surface of a sphere which shall have the point E for its centre.

Hence the curve is called a *spherical epicycloid*.

The circle A G D H, which in rolling describes the spherical epicycloid, is named the *generating circle* of that curve; and the part A C, of the circumference upon which it rolls, is called the base of the epicycloid.

When the sphere is given upon which the spherical epicycloid is required to be traced, and we know the size and position of the rolling cone which should generate this epicycloid, it will be easy, from what I have said relative to the plane epicycloid,* to find as many points of the curve, as may be necessary, and it will be evident, that what is said on spur geer, respecting

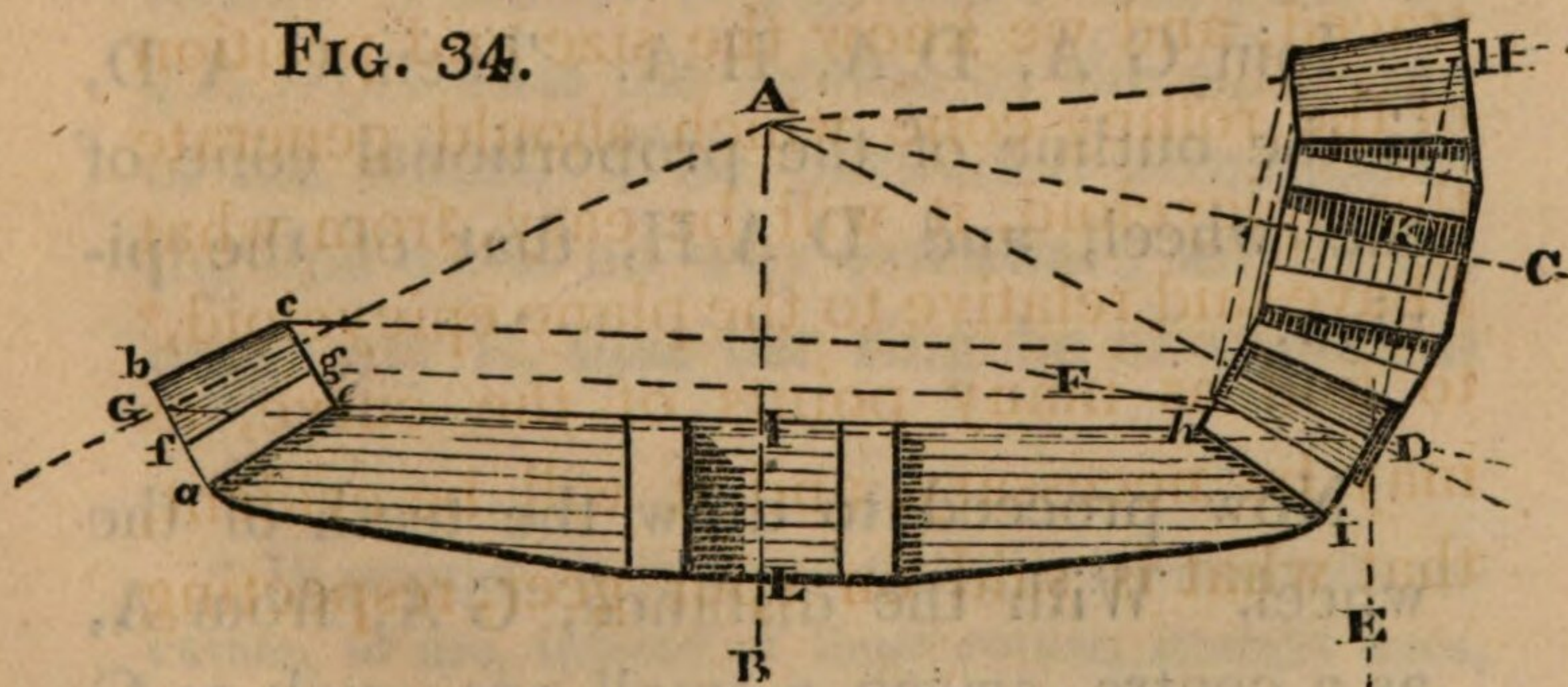
* See Chap. I. Art. 16.

the most advantageous figure of their teeth, is all applicable to bevel gear; with this difference, that the spherical is substituted for the plane epicycloid.

In order therefore to avoid tedious repetitions, I shall immediately proceed to give some account of what seems the best practical method of laying down the lines necessary to the right construction of bevel gear.

43. Having calculated the proportional diameters or pitch lines of the wheel and pinion, draw their axes, A B, A C, in the

FIG. 34.



proposed direction with respect to each other, when the wheels are in action. Parallel to $A B$, and at the distance of half the proportional diameter of the wheel, draw the line $D E$. In the same manner, draw $F D$ at the distance of half the proportional diameter of the pinion from $A C$. From the point D , where these lines intersect, draw the line $D G$, perpendicular to $A B$, and also the line $D H$, perpendicular to $C A$. Make $G I$ equal to $I D$, and $K H$ equal to $K D$. Then $D G$ is what we shall call the *principal diameter*, or the *diameter at the pitch line* of the wheel, and $D H$ that of the pinion.

Join $G A$, $D A$, $H A$. Then $G A D$, is the outline of the proportional cone of the wheel, and $D A H$, that of the pinion.

Now proceed to draw the teeth of the wheel. With the distance, $G A$, from A , as a centre, sweep a small arc, such as $G a$; at the principal diameter to their extre-

mity, set off the length of the teeth, from G to b , and draw bc tending to A .

The line bc represents the breadth of the teeth, which, according to circumstances, may be more or less; only it is to be observed, that if continued to the point A , the teeth near that point would be so small as to be of little or no use.

Describe the arc ce , concentric to ba ;* and from G to f , set off part of the required length of the tooth, from the principal diameter to the root: then draw fg tending to A , the line fg becomes the root of the tooth. Parallel to fg , draw ae , then $afge$, represents the section of the solid ring of the wheel. The particular direction of the line ae is no way essential; all that is necessary is, that the ring be of sufficient

* In practice, it is found easier, and sufficiently accurate, to use, instead of these curves, straight lines, as near as may be, in the same direction with the curves.

strength for the purpose to which it is to be applied: but patterns for cast iron wheels are usually made as represented in the plate.

FIG. 35.

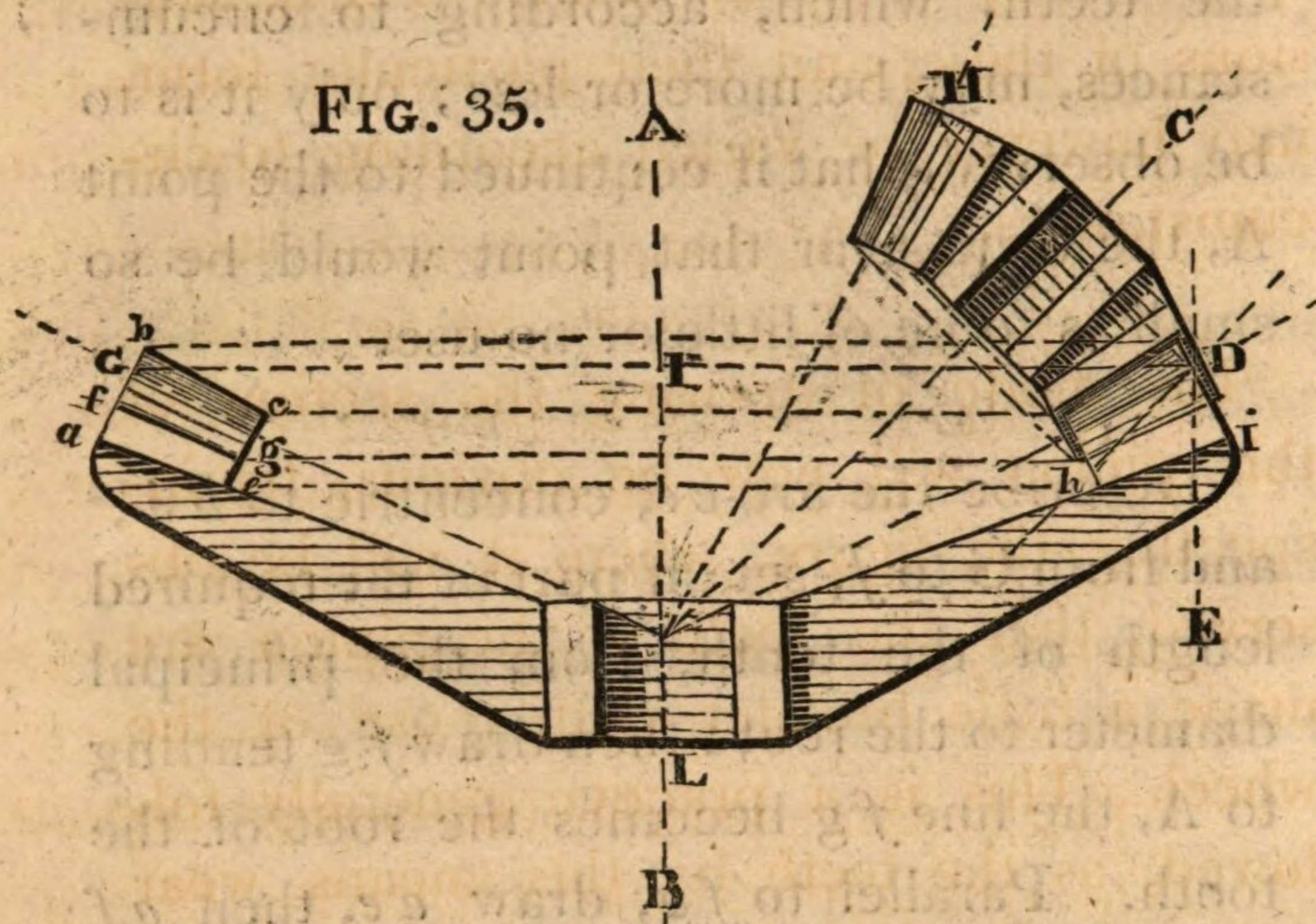
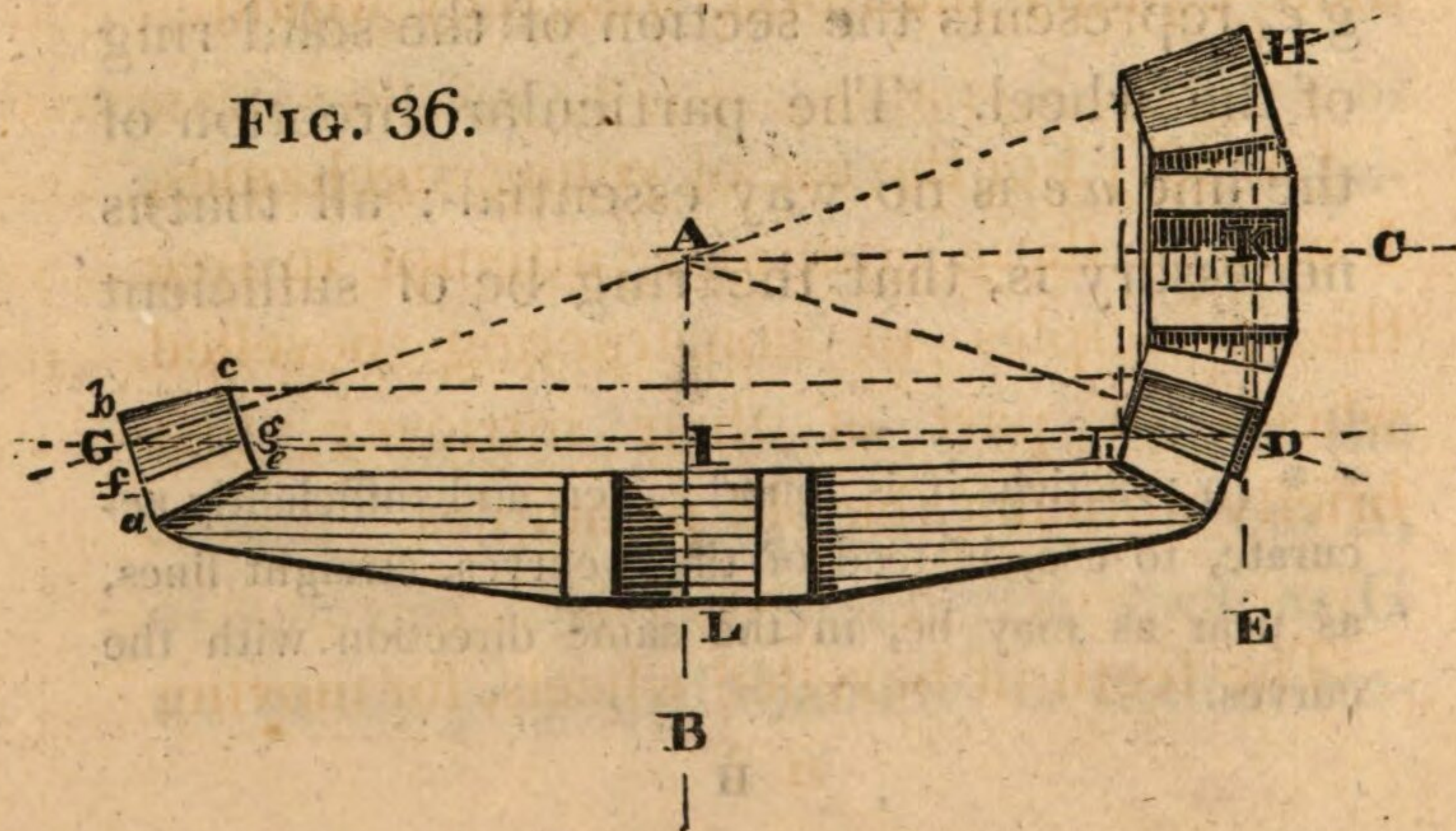


FIG. 36.



Having thus drawn a section of a tooth at G, draw in the same manner one at D, then D, I, G, L, will be the section of the wheel, in which *e, h, i, L, a*, represent the space occupied by the arms. The dimensions of these, and their particular form, may however be varied according to circumstances.

The mode of drawing the section of this pinion will now be obvious, by inspecting the figure, where it will be observed, that the teeth of the pinion are made a little broader than those of the wheel. This is a practice generally followed, as the teeth by this means wear more equally than otherwise they would.

[A. 43.] For the use of young mechanics, I will, in these additions, attempt to free the principles of constructing bevelled wheels of part of their intricacy, first briefly noticing the old principles.

The teeth of bevelled wheels, for moving

one another uniformly, may be formed according to different principles. Those which have been delivered, are, first, When the wheel drives the pinion, the acting faces of the teeth of the wheel, should be portions of a spherical epicycloid, generated by the revolution of a cone, (as described in Art. 43, Fig. 33.) of which the base is half the diameter of the pinion, and the teeth of the pinion plane surfaces directed to its centre. (Camus on the Teeth of Wheels, art. 569. Brewster, Edin. Ency. vol. xiii. p. 575.) Second, If the teeth of the pinion be staves, or formed to act as staves, then, when the wheel drives the pinion, the acting faces of the teeth of the wheel should be portions of a curve parallel to a spherical epicycloid, generated by the revolution of a cone, of which the base is equal to the diameter of the pinion. (Camus, art. 560. Brewster, Edin. Ency. vol xiii. p. 575.) To these a third may be added, that is, When the pinion drives the wheel, the most advantageous form for the acting faces of the teeth of the pinion, will

be a spherical involute of a circle, the teeth of the wheel being plane surfaces directed to its axis.

The description of these curves is not a very simple operation, nor yet adapted for application in practice; I shall therefore propose a new method, which appears to have escaped the notice of former inquirers, one which is general; spur wheels, racks, &c. being particular cases of its application. (See Fig. *E*, p. 103.)

(B. 43.) If *A B* be the axis of a pinion, and *A C* the axis of the wheel, *D E* the proportional diameter of pinion, and *D F* that of the wheel, if *C B* be made perpendicular to *A D*; then, the proper form for the acting surfaces of the teeth, may be described upon the surfaces of the cones *D E B*, and *D F C*. And since the surfaces of these cones can be spread out, or developed upon a plane surface, the form of the teeth proper for communicating

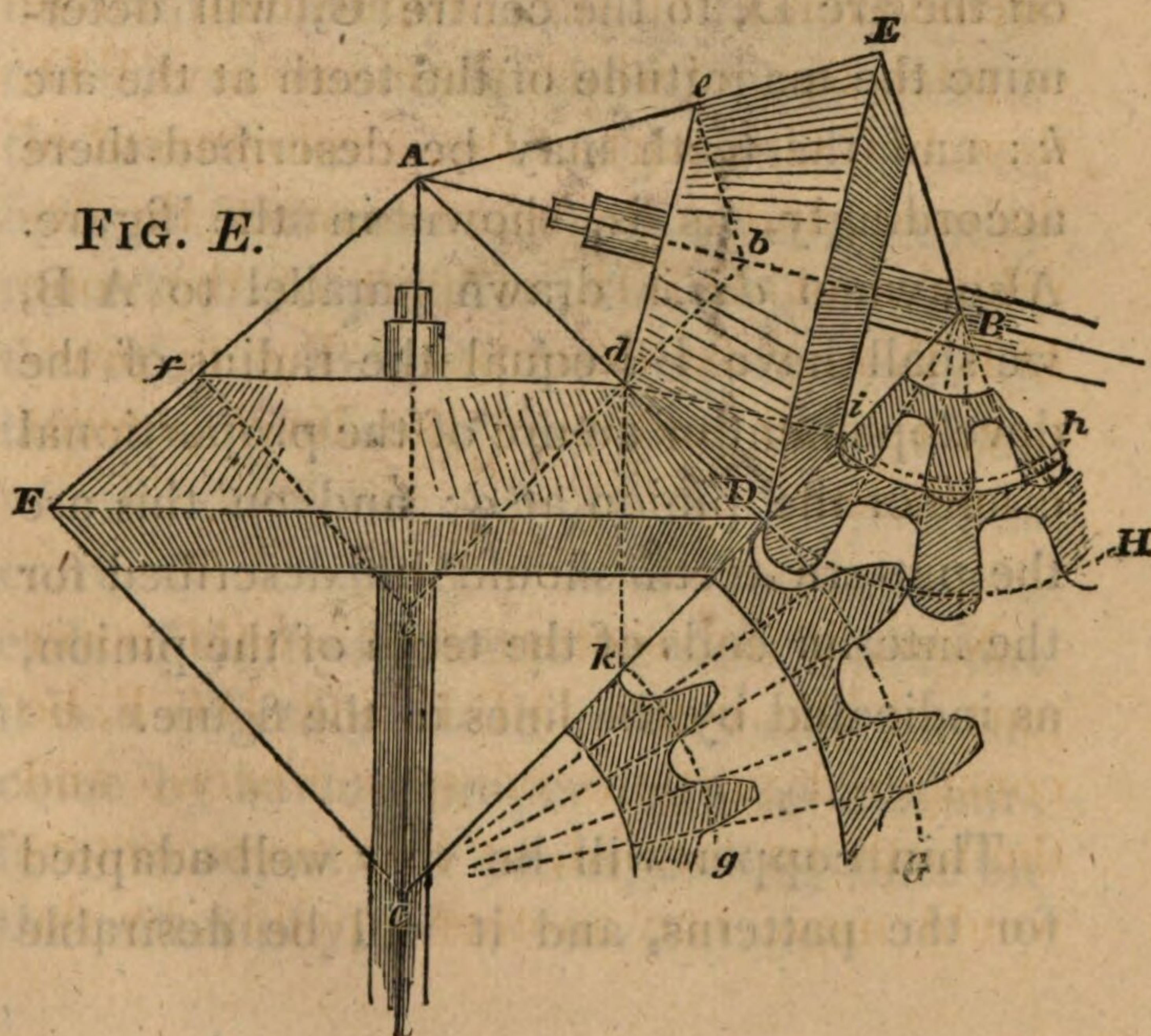
equable motion, may be drawn upon a plane.

[C. 43.] But the developement of a cone is a sector of a circle of which the radius is the slant height of the cone, and the arc equal to the circumference of the base of the cone. Thus, if the arc DG , be described with the radius CD ; this arc DG will be the developement of part of the circumference of the proportional circle or pitch line, of which the diameter is FD , then DCG is part of the developement of the cone, and in the same manner the developement DBH , of part of the cone DBE , may be described. For in practice, a small portion of the developement is sufficient

Now, if the sectors that would cover part of the cones DBE , and DCF , be considered portions of spur wheels, and the teeth be formed, so that these sectors would move one another equably, by the

methods described for spur wheels, (see Art. A 25, A 26, A 32, A 44, &c.) on sheet copper, or any other flexible body that could be applied upon the surfaces of the cones, the outline of the teeth formed by these patterns, would be such as are adapted for bevelled wheels.

[D. 43.] The breadth of the teeth d D, being settled by the same rules as for plane



spur wheels, draw cb through d , parallel to CB . Then the form of the interior end of the teeth may be found by spreading out the cones dcf , and $db e$. But this is more easily done by making dk parallel to the axis AC , and from the point k , where this line cuts CB , with the radius Ck , describe the arc kg , which is part of the developement of the proportional circle at d . Now lines drawn from the teeth on the arc D , to the centre C , will determine the magnitude of the teeth at the arc k ; and the teeth may be described there accordingly, as is shown in the figure. Also, when di , is drawn parallel to AB , we shall have Bi equal the radius of the developement of an arc of the proportional circle of the pinion at d ; and on this arc the pattern teeth should be described for the interior ends of the teeth of the pinion, as indicated by the lines in the figure.

Thin copper will be very well adapted for the patterns, and it will be desirable

that there should be two teeth upon each pattern, but more will not be necessary.

The limit of the pitch will be the same as for spur wheels, (see Art. C, 102.) and the breadth of the teeth is also to be regulated by the same rules.

The length of the teeth, the friction of them, and the peculiar advantages of the different modes of forming them, may be considered on the developed pitch lines in the same manner as if they were the pitch lines of spur wheels; consequently every remark that applies to the one, applies to the other. Indeed, the only difficulty in this construction of the teeth of bevelled wheels, consists in applying the patterns correctly to the conic surface whereon the ends of the teeth are to be described; but it is a difficulty which is very easily overcome by having proper lines on that surface, to adjust the corresponding lines on the pattern by.

Perhaps it will be of use to make a few small models of wheels, in order to fully understand the process, and apply it with certainty of success. For though it is extremely simple to those accustomed to work by developed patterns, such as carpenters, joiners, and masons,* it may not appear so, at first, to others. (Ed.)

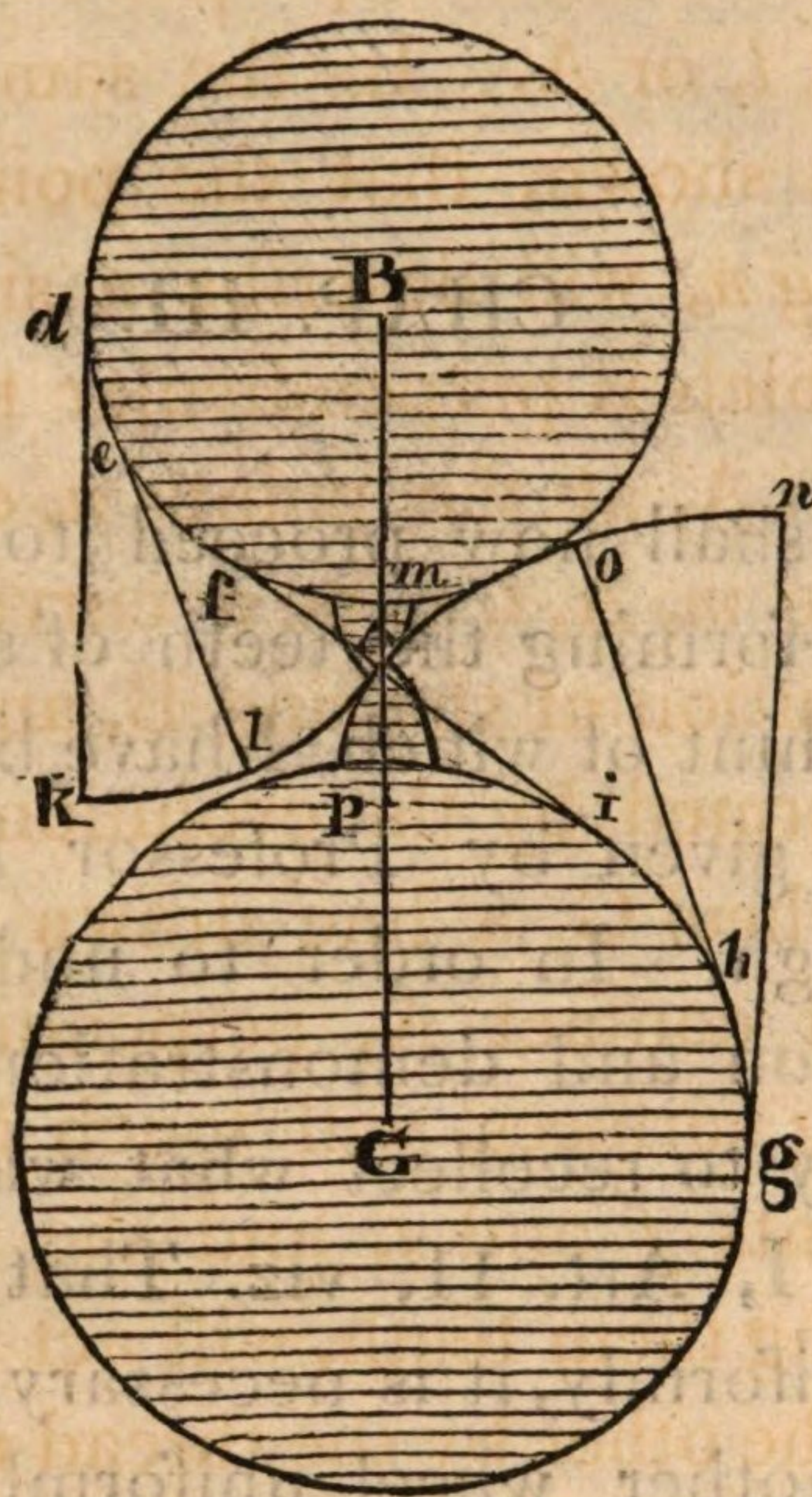
* See the art. JOINERY, Supplement to the Encyclopædia Britannica, §. 11—17; or Nicholson's Carpenter's Guide, p. 18.

CHAP. III.

44. I shall now proceed to describe a mode of forming the teeth of spur wheels, the first hint of which, I have been informed, was given by Professor Robison, of Edinburgh. In order to understand the description and demonstration, it will be necessary to recollect what was proved in Chapter I, Art. 11, viz. That if a wheel move uniformly, it is necessary, in order to move another wheel uniformly, that the form of the teeth be such as that the perpendicular from the touching surfaces *in all situations*, cut the line of centres, in the same point A, which point divides the line of centres, so that the one part A B, shall be to the other A G, as the number of teeth in the one wheel B, is to the number of teeth in the other G.

This being understood, let B and G be

FIG. 37.



the centres of the two wheels, and *d e f*, *g h i*, the rings upon which the teeth are placed; the diameters of which rings are to one another, as their number of teeth. If the thread *k d*, be lapped round the ring; as it folds up, its extremity *k*, will describe the curve *k, l, m*. It is evident, that the thread in describing the curve, is perpendicular to it; and the thread, *e*,

l, f, A , is therefore perpendicular to the curve at l , or A . In the same manner it may be shown, that the point n , of the thread, gn , will describe a similar curve, $n o p$, which is perpendicular to the thread gn , $h o, i A$, at the points n, o and A . If therefore m, A, l, k , be a curve formed by the evolution of the ring B , and $n o p$, be a curve formed by the evolution of the ring of the wheel G , a line drawn through the point of contact A , perpendicular to the touching surfaces, will touch both rings in the points f and i , and the two curves, supposing them teeth, will act as if the one pulled the other by the thread if . The line if , will be the line of action; that is, the teeth will always touch each other in a point of this line, and since this line always passes through the same point of the line of centres, $B G$, the action will be invariable; so that if the one move uniformly, the other will also move uniformly, and two weights which balance in *any one position*, will balance in all positions of the teeth.

It is obvious, that though these teeth must work, both before and after passing the line of centres, that they will work with equal truth, whether pitched deep or shallow; a quality peculiar to them, and of very great importance.

[A. 44.] The following properties of involute teeth are of most importance in wheelwork.

1. When the wheel drives the pinion, the greater part of the action will take place before the teeth arrive at the line of centres.

2. When the pinion drives the wheel, they act chiefly after passing the line of centres; and therefore involute teeth are most adapted for this case.

3. When the teeth are long, the action is very oblique, causing much unnecessary stress upon the axes; particularly when the wheel impels the pinion.

It may be remarked, that when a thread winds off from one circle to another, and these circles touch one another at the circumference, a point in the thread will describe an epicycloid; and the same epicycloid would be described by making the circle from which the thread winds off, the generating circle, the other being the base.

The length of involute teeth may be determined with sufficient accuracy by the rule, Art. 36. (Ed.)

SUPPLEMENTARY OBSERVATIONS.

45. The foregoing Essay was written several years before the publication of the Supplement to the Encyclopædia Britannica. Professor Robison has there (Vol. II. page 103, 106) described and recommended the mode which will be found, Chap. III. of forming teeth of wheels by involutes of circles. Dr. Brewster, however, in his second Edition of Ferguson's Lectures, Vol. II. page 227, observes, that this principle is not new; De la Hire having long ago considered the involute of a circle, as the last of the exterior epicycloids; which it may be proved to be, if we consider the generating straight line as a curve of infinite radius.*

Professor Robison says,† that “this form of teeth admits of several teeth to

* See Art. 53. (ED.)

† Encyclopædia Britannica, Volume XX. page 104. See also Rees's Cyclopædia, Art. Clock Movement.

be acting at the same time, (twice the number that can be admitted in M. De la Hire's method.) This, by dividing the pressure among several teeth, diminishes its quantity on any one of them, and therefore diminishes the dents or impressions which they unavoidably make on each other. It is not altogether free from sliding and friction, but the whole of it can hardly be said to be sensible. The whole slide of a tooth, three inches long, belonging to a wheel of ten feet diameter, does not amount to $\frac{1}{60}$ th of an inch, a quantity altogether insignificant.*

In the same article, this highly respectable philosopher was mistaken, in supposing, with other eminent authors, that *the mutual action of the teeth*, (when formed into epicycloids, by the method of M. Camus,) *is absolutely without friction*, and in saying, “*That one tooth only APPLIES itself to the other, and ROLLS on it, but does*

* See Art. 51. (Ed.)

not SLIDE or RUB on it in the smallest degree. This makes them last long, or rather does not allow them to wear." A very slight examination of the figures given in various parts of the preceding Essay, will, I hope, show, that the point of contact must slide from the pitch line of the conducting tooth outwards. Dr. Young, in his Natural Philosophy, Vol. II. page 183, says, that "*a form [of teeth,] without friction, is perfectly impracticable, although, for a single tooth, possible.*"

46. In the first volume of the same work, he makes the following judicious observations on our present subject:

"It has been supposed by some of the best authors that the epicycloidal tooth has also the advantage of completely avoiding friction; this is however by no means true, and it is even impracticable to invent any form for the teeth of a wheel, which will enable them to act on other teeth without friction.

“ In order to diminish it as much as possible, the teeth must be as small and as numerous as is consistent with strength and durability; for the effect of friction always increases with the distance of the point of contact from the line joining the centres of the wheels.

“ In calculating the quantity of the friction, the velocity with which the parts slide over each other has generally been taken for its measure: this is a slight inaccuracy of conception, for, as we have already seen, the actual resistance is not at all increased by increasing the relative velocity; but the effect of that resistance, in retarding the motion of the wheels, may be shown, from the general laws of mechanics, to be proportional to the relative velocity thus ascertained. When it is possible to make one wheel act on teeth fixed in the concave surface of another, the friction may be thus diminished in the proportion of the difference of the diameters to their sum.

“ If the face of the teeth, where they are in contact, is too much inclined to the radius, their mutual friction is not much affected, but a great pressure on their axes is produced ; and this occasions a strain on the machinery, as well as an increase of the friction on the axes.”*

The concluding part of these observations appears to me peculiarly applicable to the figure of teeth described in our last chapter ; for in wearing, they will be more liable than many other forms, to have *the face of the teeth, where they are in contact, too much inclined to the radius.*

* Young's Lectures, Vol. I. p. 176.

*Remarks on the Friction of Wheel Work, and
on the Forms best suited for Teeth.*

In a Letter from DR. YOUNG.

47. “ I have been considering your observations on the difference of the friction, accordingly as the teeth touch before or after the line of centres ; at first I was disposed to doubt of the fact ; but, upon more mature examination, I found that, like many other practical observations, they went beyond the scope of the doctrines of theoretical writers. I cannot however perfectly agree with you as to the explanation of the fact ; but I will state to you briefly my opinion on the subject, not having leisure at present to enter into a more ample discussion.

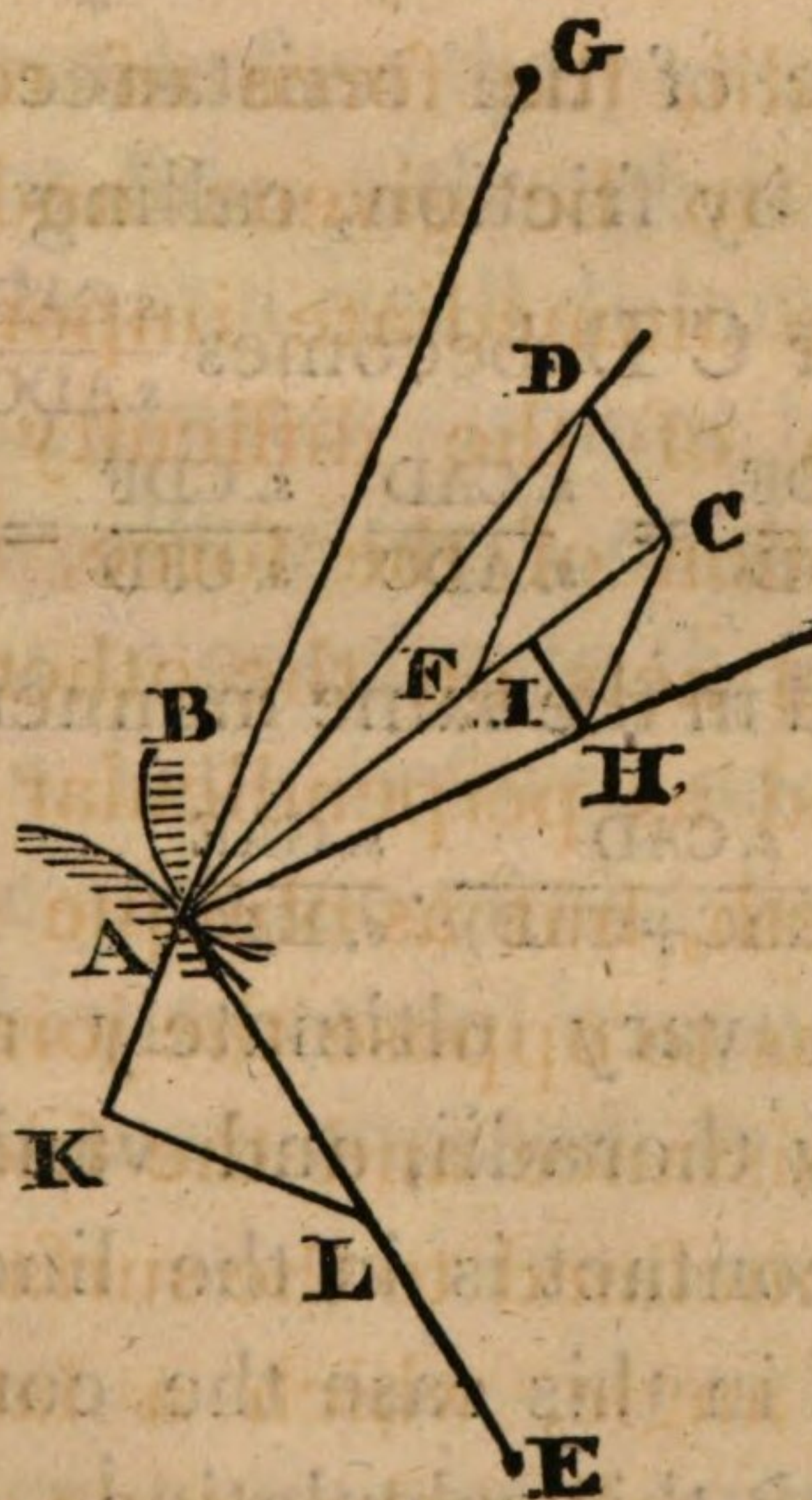
“ The magnitude of the friction has usually been estimated by the relative velocity of the surfaces concerned ; a mode of calculation, which, as I have

observed in my lecture on machinery, is only so far correct, as it shows the comparative effect of a given friction in retarding the machine. But in fact the primitive friction itself is liable to variation, according to the obliquity of the surfaces; for since the friction is nearly proportional to the mutual pressure, it will be greater or less, as the direction of these surfaces is more or less inclined to the radii, the force of rotation being supposed to be given: and, what is of still more immediate importance to the resolution of the difficulty in question, the direction of the force, by which the one wheel acts on the other, is not to be considered as perpendicular to the surface of the teeth, but as oblique to it, being so situated as to oppose the joint result of the direct resistance and the friction; that is, as being inclined to the surface in a certain constant angle, which a late anonymous writer has called the angle of repose, and which is equal to the inclination of a plane, on which one of the substances concerned

would begin to slide on the other by its gravitation.

48. " Let the tooth A impel the tooth B with the given force A C, perpendicular to the common surface of the teeth; make C A D equal to the angle of repose, then the force must act in the direction A D, and making C D parallel

FIG. 38.



to the radius A E, A D will be the actual pressure: then drawing D F parallel to

the radius A G, A F will be the effective force in the direction A C, and F C will be the loss by friction. Again, if B impel A, the angle of repose must lie on the other side of A C, and C H must be parallel to A G, and H I to A E, and the friction in this case will be I C, which is obviously less than F C.

“Hence we may easily calculate the magnitude of the resistance F C, or I C, produced by friction, calling the force A C

unity; for C D becomes $\frac{s.CAD}{s.ADC}$, and F C =

$$C D \cdot \frac{s.CDF}{s.CFD} = \frac{s.CAD}{s.ADC} \cdot \frac{s.CDF}{s.CFD} = \frac{s.CAD}{s.(CAE+CAD)} \cdot$$

$$\frac{s.GAE}{s.GAC}; \text{ and in the same manner } I C = \frac{s.CAH}{s.AHC} \cdot$$

$$\frac{s.CHI}{s.CIH} = \frac{s.CAD}{s.(GAC+CAD)} \cdot \frac{s.GAE}{s.EAC} \cdot \text{Both these}$$

quantities vary ultimately as the angle formed by the radii, and vanish when the point of contact is in the line of the centres; and in this case the common theory agrees with this calculation. When C A E is always a right angle, as in the epicycloidal tooth commonly recommended, the

friction $F C$ varies, in the different positions of the teeth, as $\frac{s. GAE}{s. GAC}$, or as $\cot. G A C$; that is, if $A K$ be made constant, as $K L$.

49. “ Since therefore it is demonstrable, that the friction is always greater in approaching the line of the centres, than at an equal distance beyond it, it must obviously be desirable that the contact should be rather after than before the passage of the teeth over that line, although it is better that it should be at a small distance before, than at a much greater distance beyond it. Hence, the impelling teeth ought to be of such a form, as to accelerate the motion of the impelled a little before, and a little more after, the passage of the line of the centres, and then to retard it again, so that the next tooth may succeed to a similar operation.

50. “ A wheel acting on a trundle, with cylindrical staves, has in this respect an advantage over two wheels with teeth,

since the curve, fitted for impelling the trundle, is adapted only to act on it beyond the line of the centres. This curve may however be formed more easily, and at the same time more advantageously, than by the method which has hitherto been recommended: for if we employ an epicycloid described by the rolling of a circle, which would just touch the internal surface of all the staves of the trundle, on the circumference of the wheel, the trundle will at first be accelerated a very little, and will then be allowed to fall back from each tooth to the succeeding one, soon after its passage over the line of the centres. The same form will also answer very well, when the trundle is to impel the wheel, although this mode of action produces a greater friction than the former.

51. “ A similar advantage may be obtained in teeth of any other form, by finishing them in such a manner as to project a very little beyond the regular outline, at the point which is intended to come into

contact a little beyond the line of the centres. Such a corrected outline may be described at once, if it be required. If the tooth is to be formed into an involute of a circle, having fitted a thread or fine wire to the circumference of the wheel, find the point of contact at the instant when the end of the wire is describing the part of the tooth which is to act at, or a little before, the line of the centres; cut off from the wheel, beyond this point, an arc equal to the distance of the centres of two adjoining teeth, and fix a pin in the tangent at the same point, that is, in the continuation of the part of the wire which is unrolled, at such a distance as just to stretch the part which is left loose by the removal of the arc: the pin thus fixed, and the remainder of the circle, will serve as bases for continuing the evolution of the wire, and the description of the tooth. The same position of the wire will show the outline of a basis proper for describing by means of a circle rolled on it, the curve which must be substituted for the form of

any epicycloidal tooth, which might have been described by causing the same circle to roll on the simple circumference of the wheel as a basis; the curved part of the tooth beginning, in this case, at the point of contact first mentioned.

“If it be objected, that in such an arrangement, the equability of the motion would be lost, and a shake would be created; it may be answered, that the inequality would be utterly imperceptible in practice. But I do not know, that the form, thus determined, would have any material advantage over teeth made as short as possible, or so cut away as not to act before the passage of the line of centres, which may easily be done in all cases, nearly in the same way as you have shown with respect to epicycloidal teeth.

“The advantage of dividing the pressure among several teeth ought not to be purchased at the expense of an increase of

friction, since the property of greater durability may be obtained, in an equal degree, by simply making the wheels thicker, without materially adding to the friction: and in fact, although the momentary pressure on each tooth may be lessened by dividing it, yet its duration is increased in the same proportion.

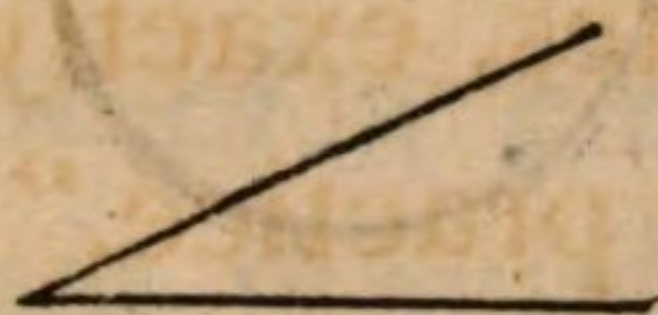
52. “ I must beg leave to observe, that the form proper for the teeth of a pinion, acting on a rack, is the involute of a circle, and not a cycloid. The cycloid would be a proper form for the teeth of the rack, if they were intended to impel the pinion.

53. “ It has been remarked that the form of the involute of a circle is not immediately deducible from the general principle of La Hire; and the remark is strictly true, since the curves, formed, according to that principle, from two contiguous circles as bases, could not act on each other without a further separation of the centres,

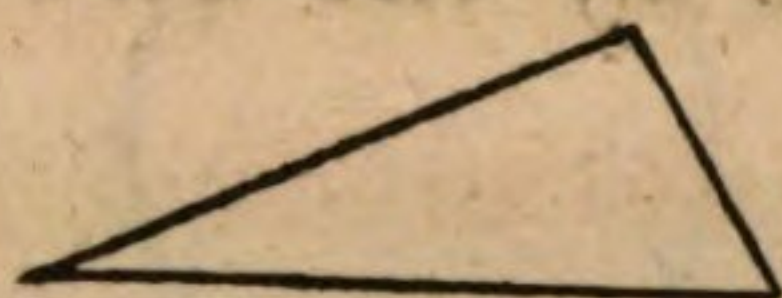
which would render the demonstration inadequate. But I have observed in the Additions to my second volume, p. x. that the principle may be extended to any other curves, as well as circles and straight lines: and if we employ an equiangular spiral, instead of a straight line, we shall have the involutes, exactly as they are recommended for practice."

SUPPLEMENTARY DEFINITIONS.

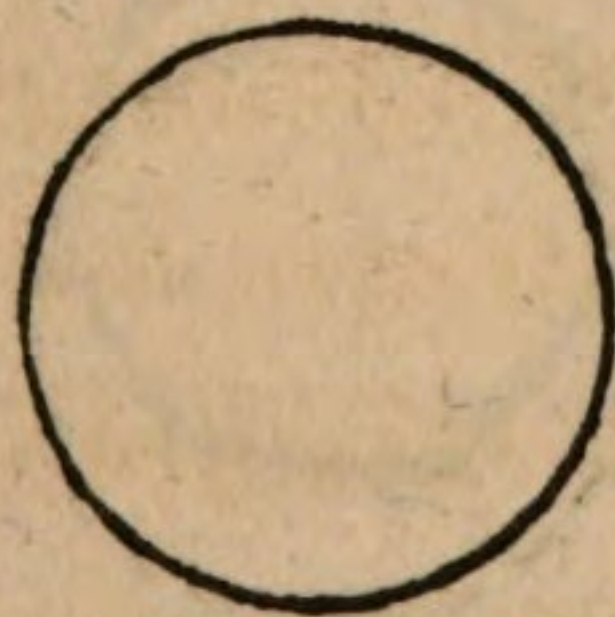
54. An angle is the inclination of two lines to one another which meeting do not lie in one line.



55. A triangle is a figure contained by three straight lines.



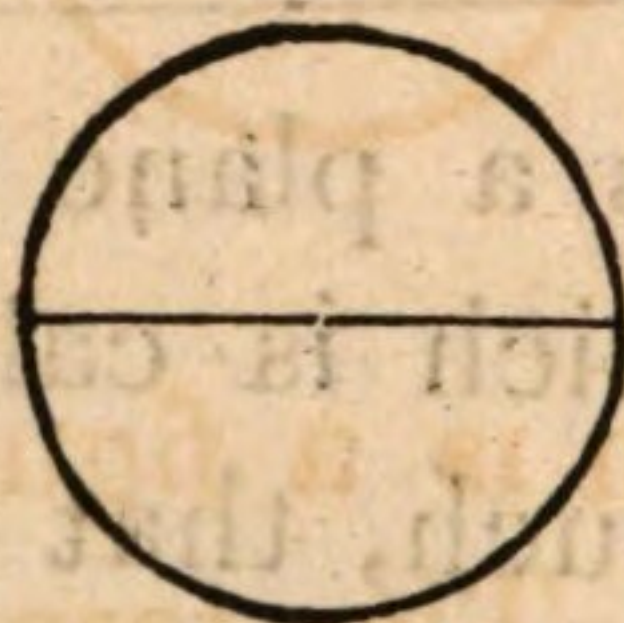
56. A circle is a plane figure contained by one line, which is called the circumference, and is such, that all straight lines drawn from a certain point within the figure to the circumference, are equal to one another, and this point is called the centre of the circle.



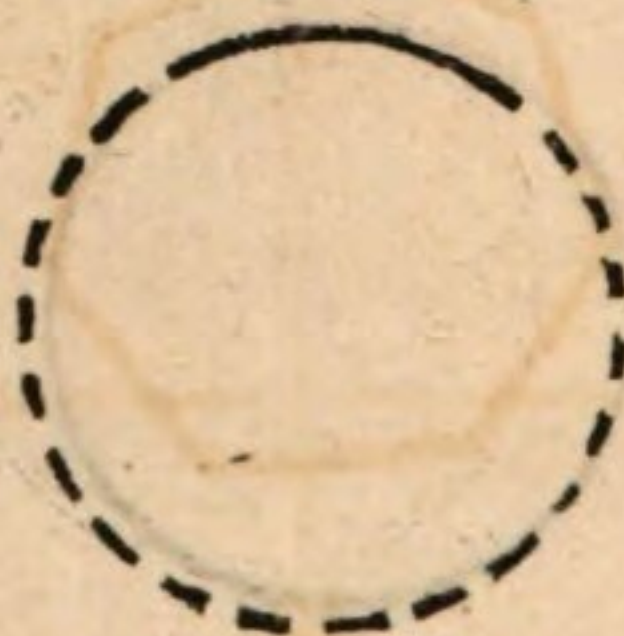
57. The radius of a circle, is a straight line drawn from the centre to the circumference. The word *radii* is used, when more than one such line is spoken of.



58. The diameter of a circle, is a straight line drawn through the centre, and terminated both ways by the circumference.



59. The arc of a circle is any part of its circumference.



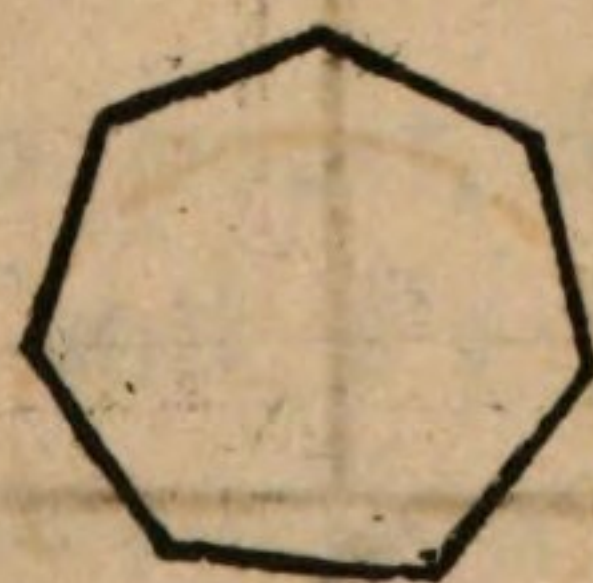
60. A chord of an arc, is a straight line joining the two extremities of the arc.



61. A tangent of a circle is a straight line, which passes through a point in the circumference without cutting it.

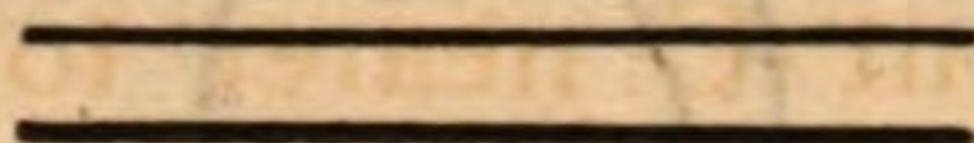


62. A polygon is a figure, having more than four sides. The term is seldom applied to figures that have less than five sides.



K

63. Parallel straight lines are such as are in the same plane, and which, being continued ever so far either way, never meet.



64. The word perpendicular is the same with square, as used by workmen.

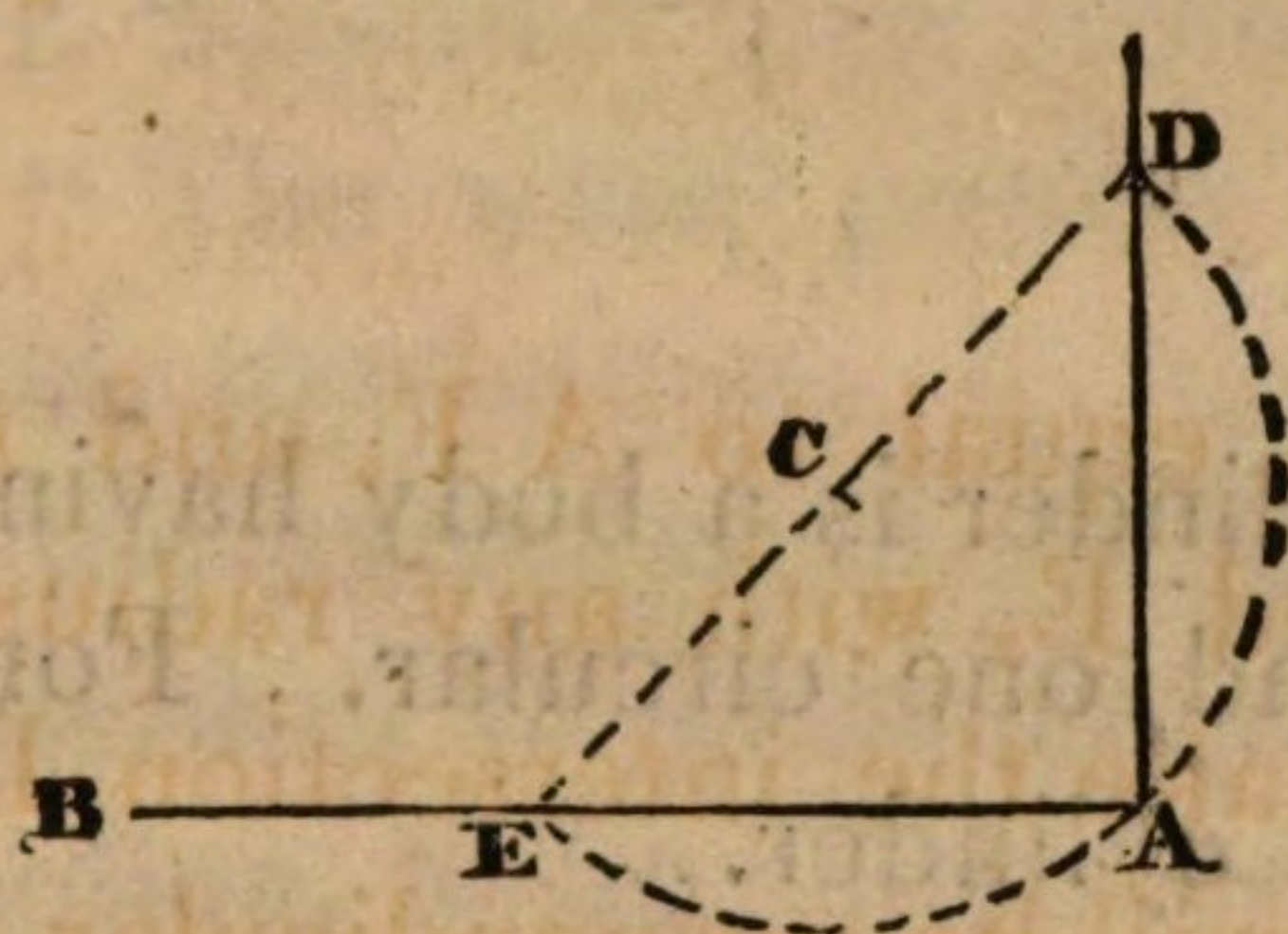
In order to draw from a given point, A, in a given line B C, another line perpendicular to it,

Take A E, equal to A F, and from the points F and E, with any radius greater than A B, make the intersection D; draw D A, which will be perpendicular to B C.



65. If it be required, from the end of a given straight line $A B$, to raise a perpendicular,

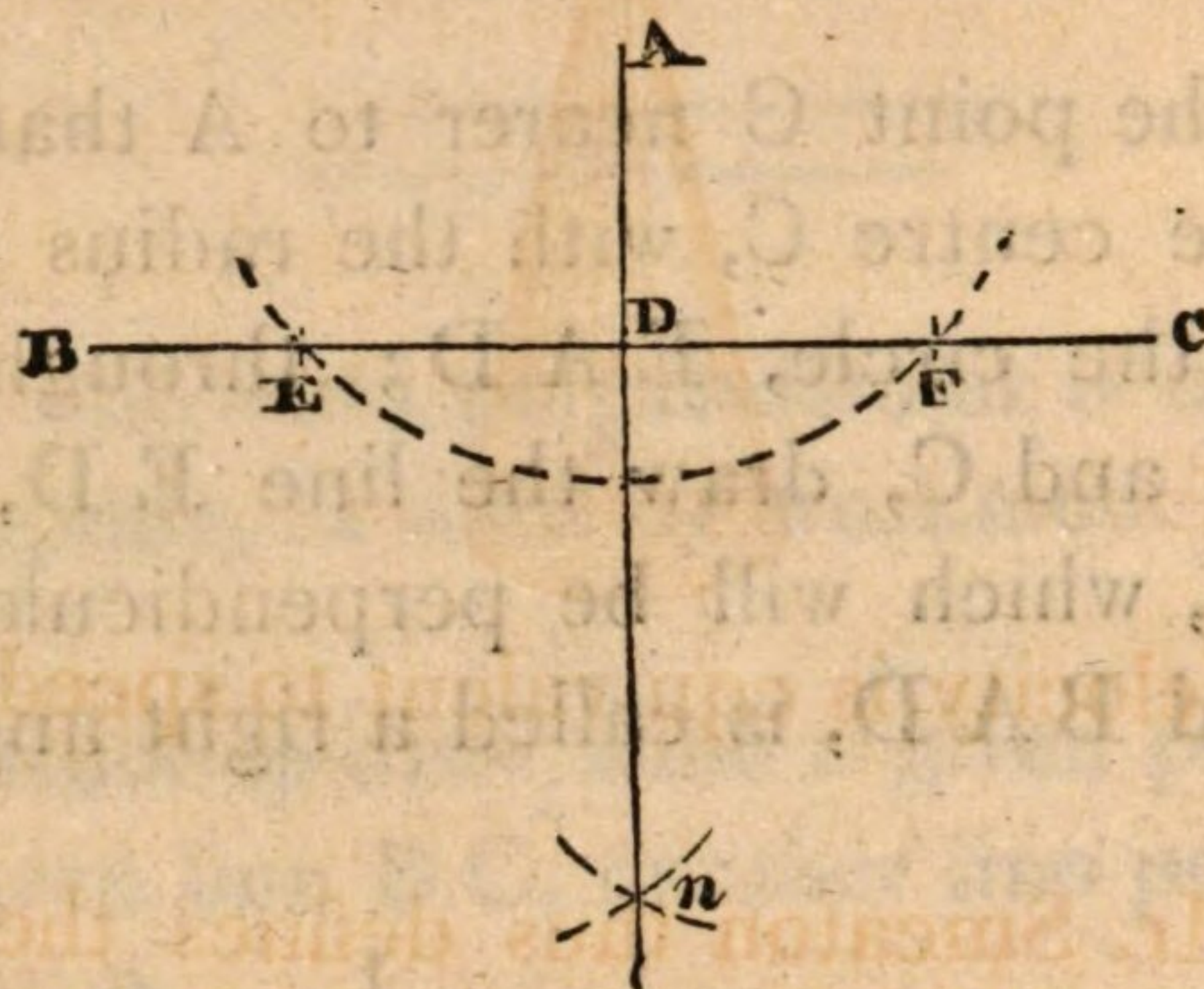
Take the point C nearer to A than B ; about the centre C , with the radius $C A$, describe the circle, $E A D$; through the points E and C , draw the line $E D$, and join $A D$, which will be perpendicular to $A B$; and $B A D$, is called a right angle



66. To let fall from a given point A , a perpendicular upon a given straight line, $B C$.

About the given point, describe a circle cutting $B C$ in E and F ; from the points E and F , make the section n , and draw the

line A D from A towards n , and A D is the perpendicular required.

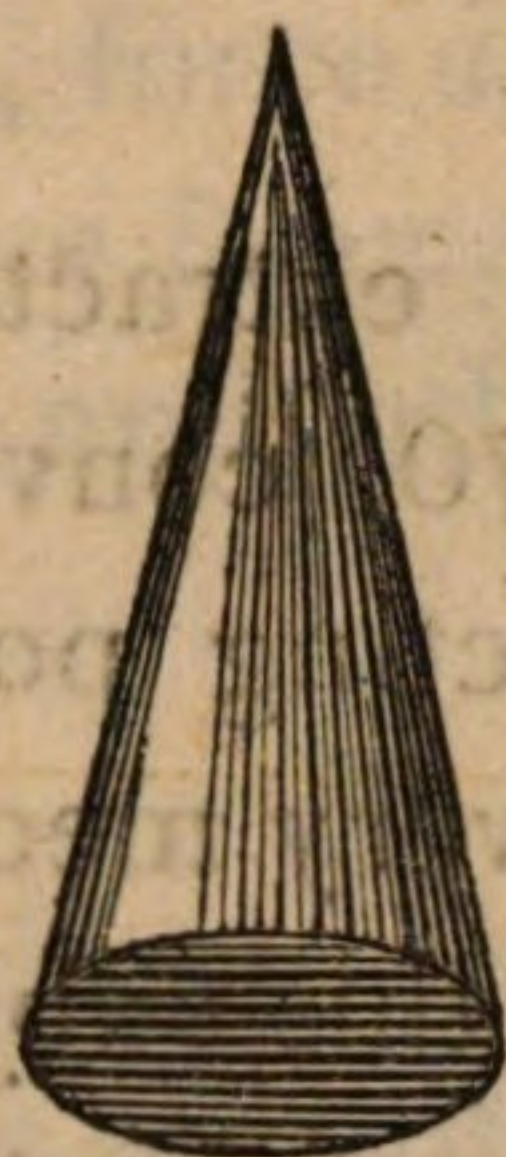


67. A cylinder is a body having two flat surfaces, and one circular. For instance, a roller is a cylinder.



68. A cone is a solid body, of which

the base is a circle, and which ends in a point.



69. Velocity is equivalent to speed.

70. Mr. Smeaton thus defines the term power: “The word power, as used in
“ practical mechanics, I apprehend to
“ signify the exertion of strength, gravita-
“ tion, impulse, or pressure, compounded
“ with motion, to be capable of producing
“ an effect; and that no effect is properly
“ mechanical, but what requires such a
“ kind of power to produce it.”*

71. A proposition is a sentence in which any thing is affirmed.

* Smeaton's Miscellaneous Papers, p. 30. See Art. A 72.

72. A corollary is an inference or deduction.

[A. 72.] The extract from Smeaton's works in Art. 70, conveys very little information respecting power; and yet it is necessary that every mechanic should have correct ideas on this subject, which will be a sufficient reason for introducing a further explanation here.

Power is the general term for that which causes motion or rest. For bodies in nature are in a state of rest, only, when the opposing powers acting upon them are in equilibrium.

But this general term, power, is divided into several particular ones according to the circumstances under which it acts.

[B. 72.] When power is, or can be, balanced at rest, it seems to be most proper to call it *force*; but, to distinguish more precisely the circumstances of its action, it

is necessary to employ the simple terms, *weight*, *pressure*, and *stress*, and also the compound terms, force of attraction, force of gravity, cohesive force, centripetal force, centrifugal force, and others of a like nature.

[C. 72.] But when a body is in motion, its power, at any instant, or at any point in its path, is usually termed *momentum*, or moving force, or quantity of motion. It is this species of power which Sir Isaac Newton makes the object of his second definition. (Mathematical Principles of Natural Philosophy, Book I.) Some writers propose to use the term energy instead of momentum, (See Edin. Rev. vol. xii. p. 130,) but there does not appear to be sufficient reason for adopting it, the other having been in a considerable degree restricted to this species of power.*

* The term energy has also been applied to the product of the mass of the body into the square of its velocity. See Dr. Young's Nat. Phil. Vol. II. Art. 347.

Here I must take the liberty of remarking, that neither the measure of momentum nor that of any other kind of power, has any relation whatever to time ; for momentum simply expresses the quantity of power in a moving body at a particular instant, without reference to the rate of accumulation, or to the effect it would produce ; and similar remarks apply to other species of power.

[D. 72.] It is further necessary, both for practical and scientific purposes, to have a term to designate, that power which is equivalent to momentum, when the velocity is uniform. Smeaton employed the term *mechanical power* for this purpose ; and since this term is sanctioned by the language of all writers on the first principles of mechanics ; and the simple machines, by means of which such power is modified to produce the desired effect, have always been called the *mechanical powers*. I think it will be found desirable to use the term *mechanical power* in preference to any

other that has been proposed. The term impetus is objectionable, because it indicates a degree of violence in the action of power, which does not agree with what takes place in the most common applications of mechanical power. And it is questionable, whether its proposer did not intend it to be a measure of effect.

I must now attempt to inform the reader, more particularly, of the circumstances to which these different modifications of power apply, and in so doing, I shall have occasion to place a most interesting department of mechanical science in a different light from what it has been regarded by my predecessors.

[E. 72.] *Force* is immediately comparable with the weight of a quiescent body. Its intensity, direction, and equilibrium, are the proper objects of that part of mechanics called statics, or hydrostatics, and aerostatics, when the body exerting force is fluid.

[F. 72.] *Momentum*, or the force of a moving body, is proportional to the quantity of matter in the body, multiplied by its velocity at that instant when the comparison is made. Its rate of increase and decrease, its direction, and equilibrium, are the proper objects of those parts of mechanics called dynamics and hydrodynamics. In fact, statics is only that particular case of dynamics when the velocity is nothing. In like manner we simplify an important part of the science of mechanics by separating all problems in which the velocity is uniform ; because in that case the length of the line the body moves over, is proportional to the velocity.

[G. 72.] *Mechanical power* then is, a particular name for the momentum of a body in uniform motion, in that case, it is proportional, to the quantity of matter in motion, multiplied by the length of the line through which it acts ; consequently, in all problems where the motions are uniform, (and there can be no difficulty in distin-

guishing such problems) this measure of power may be employed, and its motion, equilibrium, and direction, determined accordingly. Every person conversant with the management of such problems must be aware of the advantage of this mode of investigation; it applies to the motion of water-wheels, of wind-mills, of rivers, the resistance of fluids, &c. &c. and in general to the motion of machines. It has often been partially applied in considering the equilibrium of mechanical powers, (see Wood's Mechanics, prop. xxx, and xxxi.) but I am not aware of its having been previously pointed out as a general principle, with the object of forming a distinct branch of mechanics. Some writers have confounded measure of power with measure of effect so far as to suppose, that mechanical power is identical with the quantity of matter multiplied into the square of its velocity; I hope the true nature of mechanical power is here so defined as to prevent a recurrence of a like mistake.

It is much to be regretted that power has not been made the basis of all mechanical science, in the place of motion; for motion is merely an affection or mode of matter acted upon by unbalanced force. For, in the practical application of mechanics it would prevent error; and in the theory of mechanics that interesting phenomenon, *the accumulation of power*, must have been forced upon the attention of philosophers. (Ed).

ADVERTISEMENT TO THE APPENDIX.

HAVING long had it in contemplation, to make a collection of facts respecting wheels actually in use in millwork, and by arranging them, to endeavour to draw some useful practical inferences, I take the present opportunity of offering to the public the hints on that subject, which may be found in the following Appendix.

I could have wished, that time and other circumstances had allowed me to have entered more minutely into the subject. Yet hoping, that these hints, even in their present state, may lead to a fuller investigation, I trust, they will not be altogether without some advantage. As, I believe, nothing exactly of the same kind, has hitherto been published, it is hoped, that these observations will be received by the public with indulgence, and due allowance for their unavoidable imperfections.

With respect to the elementary propositions which we have to guide us in this inquiry into the proportional strength of the teeth of wheels, I shall not enter into their demonstrations. To the artisan, unacquainted with mathematics, they would be unintelligible; and the mathematician can either demonstrate them himself, or have recourse to those elementary writings where the demonstrations may be found: of these last, as being more generally accessible, I have referred to "Emerson's Mechanics," quarto edition.

Glasgow,

APRIL 5, 1808.

APPENDIX.

A PRACTICAL INQUIRY RESPECTING THE STRENGTH AND DURABILITY OF THE TEETH OF WHEELS USED IN MILLWORK.

73. HAVING treated of the forms of the teeth of wheels, I come now to consider their proportional strength with relation to the resistance they have to overcome.

I am aware, that owing to a great variety of circumstances, this subject is involved in much difficulty, and that it is no easy task to form any general rule with regard to the pitches and breadths of the teeth of wheels. I do not pretend to more than a mere approximation towards general rules ; yet, were this judiciously done, I am of opinion, that it might be useful to the millwright, who has not had leisure or

opportunity for scientific inquiries. A rule, though not absolutely perfect, is better in all cases, than to have no guide whatever.

And it is too evident to require proof, that it is essential to the beauty and utility of any machine, that the strength and bulk of its several parts, be duly proportioned to the stress or wear to which the parts may be subject.

Some general observations on the wheel work of mills, will serve greatly to simplify our inquiries on the subject.

*General Observations on the Wheel Work
of Mills.*

74. Mistaken attempts at economy have often prompted the use of wheels of too small diameter. This is an evil which ought carefully to be avoided. Knowing the pressure on the teeth, we cannot with propriety reduce the diameter of a wheel below a certain measure.

Suppose, for instance, a water wheel of 20 horses' power, moving at the pitch line with a velocity of $3\frac{1}{2}$ ft. per second. It is known, that a pinion of 4 feet diameter, might work into it, without impropriety; but we also know, that it would be exceedingly improper to substitute a pinion of only one foot diameter, although the pressure and velocity at the pitch lines in both cases would be, in a certain sense, the same. In the case of the small pinion, however, a much greater stress would be thrown on the *journeys*, (or *journals*) of the shaft. Not, indeed, on account of torsion or twist, but on account of transverse strain, arising, as well from greater direct pressure, as from the tendency which the oblique action of the teeth, particularly when somewhat worn, would have to produce great friction, and to force the pinion from the wheel, and make it bear harder on the journals. The small pinion is also evidently liable to wear much faster, on account of the more frequent recurrence of the friction of each particular tooth.

That these observations are not without foundation, is known to millwrights of experience. They have found a great saving of power, by altering corn mills, for example, from the old plan of using only one wheel and pinion, (or *trundle*,) to the method of bringing up the motion, by means of more wheels and pinions, and of larger diameters and finer pitches.

The increase of power has often by these means been nearly doubled, while the tear and wear has been much lessened; although it is evident, the machinery, thus altered, was more complex.

The due consideration of the proper communication of the original power, is of great importance for the construction of mills on the best principles.* It may easily be seen, that in many cases, a very great portion of the original power is ex-

* See Essay II. Chap. I. Sect. ii.

pended, before any force is actually applied to the work intended to be performed.

Notwithstanding the modern improvements in this department, there is still much to be done. In the usual modes of constructing mills, due attention is seldom given to scientific principles. It is certain, however, that were these principles better attended to, much power, that is unnecessarily expended, would be saved. In general, this might be in a great measure obtained, by bringing on the desired motions in a gradual manner, beginning with the first very slow, and gradually bringing up the desired motions, by wheels and pinions of larger diameters. This is a subject which should be well considered before we can determine, in any particular case, what ought to be the pitch of the wheels. In the case above alluded to, where the supposition is a pinion of 4 feet diameter, or of 1 foot diameter; it is obvious, that the same pitch for both would not be prudent. That for the small pinion,

ought to be much less than that which might be allowed in the case of the larger pinion. It is also equally obvious, that the breadth of the teeth, in the case of the small pinion, ought to be much greater than that in the case of the larger pinion.

75. It is evident, however, that although great advantage may often be derived from a fine pitch, that there is a limit in this respect, as also with regard to the breadth.* We shall endeavour to find some trace of this limit in what follows; and that we may the better do this, we shall call in the aid of propositions, which are true with respect to pieces of timber, or metal, subjected to ordinary cases of pressure. It is allowed, that they cannot here, in strictness, be *demonstrated*, as applicable to wheelwork. Yet they will, for want of better light, serve at least to prevent any material practical error with regard to the strength of the teeth of wheels.

* See Additions to Rules for Strength of Teeth.

For it is to be remembered, that we are not so much here in search of truths of curious or profound mathematical speculation, as of that kind of evidence of which the subject admits, and which may be sufficiently satisfactory for any practical purpose

I would have it, however, understood, that we suppose the diameters made sufficiently great to prevent the evils which we have already noticed, and in the annexed table, (Art. 86,) are some examples in actual use, which have been found in practice sufficiently durable. I would particularly recommend attention to those of Messrs. Boulton and Watt, whose most extensive practice, as well as scientific knowledge, renders their work a model well worthy the attention of millwrights.

As cast-iron pinions are now generally used, and as the teeth of the pinion are most subject to wear, I think we are safe

in the present inquiry, in considering them all as cast iron.

The laws to which I have alluded in this investigation are these:

*Principles of proportioning the Strength of
Teeth of Wheels.*

PROPOSITION I.

76. “*The Strength of any Piece of Timber, or Metal, whose Section is a Rectangle, is in direct Proportion to the Breadth, and as the Square of the Depth.*” *

Hence may be inferred, that the strength of teeth of wheels, moving at the same velocity, and under the same circumstances, is directly in proportion to their breadth, and as the square of their thickness.

* See Emerson, Prop. 67.

Thus, for example, if we double the breadth, we only double the strength; but if we double the thickness, in other words, double the pitch, keeping the original breadth, we increase the strength four times.

For although when wheels are working accurately, the strain is at the same time divided over several teeth, yet as a very small inaccuracy, or even the interposition of any small body, such as a chip of wood or stone, throws the whole stress upon a single tooth in practice; therefore, and in order to simplify this case, we may consider the strength of a single tooth, as resisting the pressure of the whole work.

But as the length of teeth commonly varies with the pitch, this circumstance must be taken into account, and the most simple view we can take of it seems to be, that of having the strain of each tooth, thrown all to the outward extremity; we

have then the following proposition to guide this part of our inquiry.

PROPOSITION II.

77. *If any Force be applied laterally to a Lever or Beam, the Stress upon any Place, is directly as the Force and its Distance from that Place.**

PROPOSITION III.

78. *The Pitch being the same, the Stress is inversely as the Velocity.†*

For example—if the pitch lines of one pair of wheels be moving at the rate of 6 feet in a second, and another pair of wheels, in every other respect under the same cir-

* See Emerson, Prop. 69.

† See Emerson, Prop. 119, Rule 8.

cumstances, be moving at the rate of 3 feet in a second, the stress on the latter is double of that on the former.*

* 'This proposition is true only in the wheels of the same machine. To render it universal, the first movers of all machines must be reduced to the same standard; which may be done as follows, where the horse power is supposed to be the standard.

If P be the power of the first mover, in any machine, and V its velocity; also, let v be the velocity of any part on which it is necessary to determine the stress.

Then, as $v : V :: P : \text{stress} = \frac{PV}{v}$, (Wood's Mechanics, Prop. XXXI.)

Now taking the same value of the horse power as Mr. Buchanan has used throughout these Essays; that is, 200lbs. moved at the rate of $3\frac{2}{3}$ feet per second, and using H to represent the number of horses, we have

$$\frac{PV}{v} = \frac{200 H \times 3\frac{2}{3}}{v} = \frac{733\frac{1}{3} H}{v}.$$

By using $\frac{740 H}{v}$ = the stress, we get rid of the fractions, and have a sufficiently accurate measure of the force. Hence, if the power of a machine be equal to

We shall confine our attention for the present to wheels having cast-iron teeth; and, in order to take experience as our guide, several examples in the annexed tables, actually in use, are selected.

The pitch, velocity, and strain, are all stated; the strain is measured by the *horses' power*, at which the resistance is valued.

any number H of horses, the stress at any pitch line of which the velocity is v feet per second will be $= \frac{740 H}{v}$.

In a like manner, the stress at the surface of any journal or shaft may be found.

But, in those cases where the same first mover gives motion to different trains of machinery, the stress, at any part of any one of them, should be measured by the greatest number of horses' power necessary to perform the work assigned to that train. That is, if H be the number of horses that could perform the work, then $\frac{740 H}{v}$ = the stress at any point in the train moving with the velocity v . (Ed.)

Horse's power is a term now in general use, to express the force required, in order to drive any kind of mill, and it may be proper here to give some further account of it.

Horse's Power.

79. Although horses are not all of one strength, yet there is a certain force now generally agreed upon among those who construct steam engines, which force is denominated a *horse's power*, and hence, steam engines are distinguished, in size, by the number of horses' power to which they are said to be equal.

The measure of a mechanical effect equal to a horse's power, has been much disputed : this I believe to be a matter of little consequence, if the measure be generally understood, since there is no such thing as bringing this into any real measure. Some horses will work double of others, and horses of one country will work

more than those of another. Desaguliers's measure is, that a horse will walk at the rate of $2\frac{1}{2}$ miles per hour, against a resistance of 200lbs,* and which gives, as a number for comparisons, 44000; that is, the raising of 1lb. 44000 feet in a minute; or, what amounts to the same, the raising of 44000lbs. 1 foot in a minute.

Emerson's measure is the same as Desaguliers's, (see Emerson's *Mechanics*, p. 178,) and Mr. Smeaton's result is 22916lbs. under the same circumstances.†

Mr. Watt has found, from repeated experiments, that 33000lbs. 1 foot per mi-

* When working 8 hours a day, (Desaguliers's *Course of Experimental Philosophy*, Vol. I. p. 241.) $2\frac{1}{2}$ miles per hour is equal to 220 feet per minute, or $3\frac{2}{3}$ feet per second. (Ed.)

† Reports, Vol. I. p. 229. But Desaguliers gives the immediate power of a horse, Smeaton the effect of that power applied to raise water: hence the friction of the machinery should be added to Smeaton's horse's power. (Ed.)

nute, was the average value of a horse's power; but, I believe that his engines were calculated to work equal to 44000lbs. 1 foot per minute.

But, that he allows only 33000 in his calculations, applied to mills, considering the difference as being lost in the friction of the engine itself.

80. It is common in practice, to reckon, that it requires one horse's power to drive 100 spindles with preparation of cotton water twist.

81. 1000 spindles with preparation cotton mule yarn.

82. 75 spindles with preparation flax yarn.

I beg leave here to make the following extract, on the subject of animal force, from Dr. Young's Natural Philosophy, Vol. II. p. 165.

83. "In order to compare the different estimates of the force of moving powers, it will be convenient to take a unit which may be considered as the mean effect of the labour of an active man, working to the greatest possible advantage, and without impediment; this will be found, upon a moderate estimation, sufficient to raise 10 pounds 10 feet in a second, for 10 hours in a day; or to raise 100 pounds, which is the weight of 12 wine gallons of water, 1 foot in a second, or 36000 feet in a day, or 3,600,000 pounds, or 432000 gallons, 1 foot in a day; this we may call a force of 1, continued 36000".

84. Immediate force of men and horses,
without deduction for friction.

	Force.	Conti- nuance.	Day's Work.
“A man of ordinary strength can turn a winch, with a force of 30 pounds, and with a velocity of $3\frac{1}{2}$ feet in 1" for 10 hours a day.”—Desaguliers	1.05	10h.	1.05
“Two men working at a windlass, with handles at right angles, can raise 70 pounds more easily than one can raise 30.”—Desaguliers	1.22		1.22
“For a short time, a man may exert a force of 80 pounds, with a fly, when the motion is pretty quick.”—Desaguliers	3.	1"	
“A horse can draw, with a force of 200 pounds, $2\frac{1}{2}$ miles an hour for 8 hours in the day	7.33	8h.	5.87
“With a force of 240, only 6 hours.”—Desaguliers	8.8	6h.	5.28

85. Performance of men and horses by machines.

	Force.	Continuance.	Day's Work.
"A man can raise, by a good common pump, a hogshead of water 10 feet high in a minute, for a whole day."—Desaguliers.....	.875		.875
"By means of pumps, a horse can raise 250 hogsheads of water 10 feet high in an hour."—Smeaton's Reports *	3.64	8h.	

[A. 85.] The power of men and horses to move machines, has very frequently been made a subject of investigation by writers on mechanics; but yet it appears possible to consider it in a different manner, which will furnish results from principles somewhat more strictly practical than those which have hitherto been made the basis of calculation.

1. It is almost always a necessary con-

* Vol. I. p. 229.

dition, that the moving power of a machine be sensibly uniform, and consequently the power which moves it, should be uniform, or of that kind termed mechanical power, see Art. G. 72.

2. Let $P D$ be the mechanical power of a horse, which can be continued during a whole day, and also day after day, without exhausting its strength. Then, since the exertion of the muscles will be constant, the product $P D$ will be a constant quantity, for the degree of exertion will not be altered by altering either D , the distance passed over in a second, or the force P . But, let D be the distance when the force P is the least possible corresponding to the manner of action the machine requires, or in other words, when the power to move the machine to produce useful effect would be nothing. And let $p d$ be the mechanical power, when the distance passed over in a second is d . Then, $P D = p d$. and, $d(p - P) =$ the mechanical power exerted on the machine, which is to be the greatest

possible. But $P = \frac{p d}{D}$, therefore $p (d - \frac{d^2}{D})$
 = a maximum.

Now it may be shown by the principles of maxima, &c. that this quantity is a maximum when $d = \frac{1}{2} D$.

Therefore, a man or a horse acts with the greatest advantage on a machine when he moves with half the velocity he could continue at, were the effective resistance of the machine nothing.

That portion of the mechanical power which is efficient in impelling the machine will be $\frac{1}{2} P D$. For since $d = \frac{1}{2} D$, $P = \frac{p d}{D} = \frac{1}{2} p$, or $2 P = p$; hence $d (p - P) = \frac{1}{2} D (2 P - P) = \frac{1}{2} P D$.

The force P , and the distance moved through D , will each vary in the same man or horse according to the manner of applying the force; but their product will be nearly a constant quantity. In any case, one

of these quantities may be determined from experience, and in many instances both of them, and that one may always be supplied by calculation which cannot be found by experience.

When the effective force is nothing, it must not be understood that a man or a horse is acting against a force equal to his own weight, in any case whatever; for as has been observed by Dr. Young, (Vol. I. p. 132, and 212,) in walking, the resistance overcome is not exactly comparable with mere weight, and the same may be remarked on other modes of exerting force.

When a man ascends vertically, his velocity is reduced to about one half of his horizontal velocity, indicating that he acts against a double resistance; therefore when a man ascending a ladder, carries a load, the maximum effect will take place when his ascending velocity is about one fourth of the velocity he can walk horizontally without a load.

A man of ordinary strength will not be able to walk, unloaded, at a quicker rate than $3\frac{1}{2}$ miles an hour, if this exertion is to be continued for 10 hours every day. Indeed, those who examine the subject with a view to a fair average, will find this to be about the extreme velocity that can be continued, without injury, for any considerable time.

According, therefore, to our investigation, a man ought to move with half this velocity to produce a maximum effect; that is, at the rate of $1\frac{1}{2}$ miles an hour, which is about $2\frac{1}{2}$ feet per second.

But this supposes the whole load to be the useful effect, whereas part of it must consist of the apparatus employed to carry it, or the friction of an intermediate machine, or other circumstances of a like nature. About one fifth of the velocity may be considered equivalent, at an average, to the force lost in friction, &c. in all cases, in many it will exceed one fifth.

Hence the maximum of useful effect will take place when the velocity is 2 feet per second, or about 11 furlongs an hour, continued for 10 hours each day.

Smeaton is said to have made numerous comparisons, from which he concluded that the mechanical power of a man is equivalent to 3750 lbs. moving at the velocity of one foot per minute;* and taking this average to be near the true one, as I have reason to conclude it is, we have $\frac{3750}{2 \times 60} = 31.25$ lbs. Therefore, we make the average mechanical power of a man 31.25 lbs. moving at the velocity of 2 feet per second, when the useful effect is the greatest possible; or half a cubic foot of water raised two feet per second; a very convenient expression for hydrodynamical inquiries.

If a man ascend a vertical ladder, according to a preceding remark, (p. 163.) the

* Art. Water, Rees's Cyclopædia.

velocity which corresponds to the maximum of useful effect will be 1 foot per second, and the load double that which he carries horizontally ; consequently the average of useful effect is 62.5 lbs. raised one foot per second.

Bricklayers' labourers in London ascend ladders with a load of about 80 lbs. besides the hod ; sometimes at the rate of one foot per second, but more frequently about 9 inches per second.

Ascending stairs is more fatiguing to the muscles of the legs than ascending a ladder ; and therefore the useful effect is less, till a person has become accustomed to this kind of labour. And it is also to be observed that the space moved over is increased, unnecessarily, except where the horizontal distance is part of the path over which the load is to be moved.

We ought not to be surprised at the opposite conclusions here obtained, from

those of other theoretical inquirers, when the data they have proceeded from are considered. For their data have been the extremes of force and velocity, without any knowledge of the true laws which connect them.

3. The force of a horse, is at an average, about equal to that of six men, according to various estimates; and the rate of travelling about the same, perhaps rather less than that of a man, when his exertion is continued for 8 hours; consequently the velocity corresponding to the maximum effect, will be about $2\frac{1}{2}$ feet per second. Whence, the average mechanical power of a horse may be estimated at $187\frac{1}{2}$ lbs. moving with a velocity of $2\frac{1}{2}$ feet per second, or 3 cubic feet of water raised $2\frac{1}{2}$ feet per second. The day's work being 8 hours.

This estimate of the power of a horse is equal to 28,125 lbs. raised one foot per minute; nearly a mean between Watt's

and Smeaton's. But since our author has employed that of Desaguliers, it was not very easy to make a change in this work; and still more objectionable to employ two measures of different values.

French writers use as a dynamical unit a given measure of water raised through a given space; and the Americans use a similar measure. In my opinion, it is preferable to make a horse's power the dynamical unit; because, a practical man has a more correct idea of the quantity of mechanical power expressed by this unit, than he can have of any one less frequently under his observation; besides, it is a power often employed to move machines; and therefore, is a familiar measure of comparison. (Ed.)

Table of Pitches of Wheels in actual Use in Mill-work.

Kind of Machine.	Horses' power.	Pitch in inches.	Breadth of teeth in inches.	Wheel.			Pinion.			Breadth proportionate to 10 horses' power, and present velocity.	Present velocity per second, in feet.	Breadth in inches proportionate to 10 horses' power, at 3 f. p. second, that is, reducing all the examples to the same denom.
				Teeth.	Revs. per minute.	Diameter.	Teeth.	Revs. per minute.	Diameter.			
Water-wheel, ¹ A	10	4 $\frac{1}{8}$	5 $\frac{1}{2}$									5.5
Water-wheel, ² B	30	3	10 $\frac{1}{4}$	304	3 $\frac{1}{7}$	24	3	318.47		3.41	3.95	4.489
Water-wheel, C	15	3	6	204	4 $\frac{1}{2}$	16..2	44	20	3..6	4.	3.8	5.06
Water-wheel, ³ D	5 $\frac{1}{2}$	3	4	207		16..5 $\frac{1}{4}$	50		3..11 $\frac{5}{8}$	7.27	3.	7.27
Horse-mill, ... E	1	2 $\frac{1}{4}$	4	91	3	6..0 $\frac{1}{2}$	22	12.9	1..5 $\frac{1}{2}$	40.0	.949	12.65
Horse-mill, ... F	1	2 $\frac{1}{4}$	4 $\frac{1}{2}$	91	3	6..0 $\frac{1}{2}$	20	13..13	1.4	45.0	.949	14.23
Steam Engine, by B. & W. G }	24	3 $\frac{1}{4}$	6	96	19	8..0	42	43.32	3..6	2.5	7.95	6.625
Steam Engine, by B. & W. H }	46	3	8	152	17 $\frac{1}{2}$		54	50		1.7	11.	6.2
Steam Engine, by B. & W. I }	32	3	6	116	19	8..10				1.87	8.78	5.47
Steam Engine, by B. & W. K }	14	3	5	64	25	5..1	29	55	2..4	3.57	6.65	7.91
Steam Engine, L	20	2 $\frac{1}{2}$	5	90	18	5..11	38	42.63	2..7	2.5	5.57	4.64
Ditto, ditto, ⁴ M	10	2 $\frac{1}{4}$	5 $\frac{3}{4}$	77	25	4..7 $\frac{1}{8}$	40	48.5	2..4 $\frac{5}{8}$	5.75	6.2	11.88
Ditto, ditto, ⁵ N	6	2 $\frac{1}{4}$	5 $\frac{1}{4}$	60	28	3..7	27	62.6	1..7 $\frac{3}{8}$	8.75	5.25	15.31
Ditto, ditto, ⁶ O	4	2 $\frac{1}{4}$	4 $\frac{3}{4}$	48	32	2..10 $\frac{1}{5}$	25	61.11	1..6	11.87	4.8	18.99
Ditto, ditto, ... P	2	2	4 $\frac{1}{2}$	62		3..6			2..	3.75	8.8	11.
Ditto, ditto, ⁷ Q	10	1 $\frac{7}{8}$	6	77	25	3..10	40	48.5	1..11 $\frac{1}{8}$	6.	5.	10.
Ditto, ditto, .. R	12	1 $\frac{3}{8}$	3	66	44	2..8	48	60.5	1..9	2.5	5.99	4.95

¹ Has been 16 years at work. Teeth much worn.

² Has been 16 years at work. This gearing was found rather too narrow for the strain, as it is wearing much faster than the rest of the wheels in the same mill.

³ The only defect in this gearing, which has been 16 years at work, is the want of breadth in the spur-wheel and pinion: they ought to have been 6 inches or more, as they will not last half so long as the bevel-wheels and pinions connected with them.

⁴ This is a better pitch for the power than Q.

⁵ This wheel has wooden teeth, and has been working for three years past.

⁶ Ditto.

⁷ This pitch has been found to be too fine.

*Explanation of the Table of Wheels in
actual Use in Millwork.*

86. The wheels are all reduced to what may be called one denomination.

First—By proportioning their breadths all to what they should be to have the same strength, if the resistance were equal to the work of a steam engine of ten horses' power.

Secondly—By supposing their pitch lines all brought to the same velocity of 3 feet per second, and proportioning their breadths accordingly. I have chosen this particular velocity of 3 feet per second, because it is the velocity very common for overshot water wheels.

Such cases as appear to have worn too rapidly, are marked, which may tend to discover the limit in point of breadth.

Column	1	Contains the Horses' power.
_____	2	_____ Pitch in inches.
_____	3	_____ { Breadth of teeth in inches.
_____	4	_____ { Number of teeth of wheel.
_____	5	_____ { Revolutions of wheel per minute.
_____	6	_____ Diameter of wheel.
_____	7	_____ { Number of teeth of pinion.
_____	8	_____ { Revolution of pinion per minute.
_____	9	_____ Diameter of pinion.
_____	10	_____ { Breadth proportionate to 10 horses' power, and at the present velocity.
_____	11	_____ { Present velocity per second in feet.
_____	12	_____ { Breadth in inches proportionate to 10 horses' power, at 3 feet per second. That is, all the cases reduced to the same denomination.

*Observations on the Table of Wheels in actual
Use in Millwork.*

87. Having reduced the examples in the table, in the manner already described, to one denomination, the results approach nearer, considering all circumstances, than could have been expected.

1st, In two of the cases, viz. B and D, it appears, that the wheels were rather too narrow for their work. These, however, have been working about sixteen years, and may yet continue for a long time.—B is 4.489 inches in breadth, when reduced to 10 horses' power, at 3 feet per second.—D is 7.27 inches in breadth. The pitch of both is three inches.

Three inches being a pitch in very general use for the first motion of mills, could the proper breadth for this pitch be ascertained, it would serve as a very useful standard.

Rules to be of practical use, must be easy of remembrance, as well as easy of application.

Let us, therefore, assume a simple standard, and try how far it will bear the test of experience; for, as Du Buat justly observes, "It is an excellent method, in the research of obscure difficult truths, to suppose a theory pre-existent, founded upon the most probable principles, from which may be determined, the choice of some direct experiments, proper to evince the fallacy or accuracy of the principles proposed."

The actual cases in the table, may be considered as satisfactory experiments. Let us, therefore, try the following simple rule, and compare some of its results with those cases.

88. Rule I. for pitch of three inches, when the velocity is three feet per second, at the pitch line.

Make the teeth as many inches broad as the number of horses' power which it has to resist.

For example, for nine horses' power, make the teeth nine inches broad.

89. Taking this example then as a point from which to set off, and supposing the same breadth of *nine inches* constant, we shall, in the first following table, state various pitches, and (by Proposition I.) shall first square these pitches, to find the number of horses' power, equal to the strength and durability of the pitch, when the teeth are all of one length. The strength thus found, will be inserted in column Y.

But, as the lengths generally vary as the pitches, taking the same point, (three inches pitch, nine inches broad) from which to set off, we shall diminish the value of the strength, as we ascend, and increase as we descend, agreeably to Proposition II. The results will be found in column Z.

But as in this investigation durability is of equal importance with strength, perhaps the true proportion may be somewhere between the results in the columns Y and Z.

*Description of the six following Tables of
Pitches.*

In all the following tables, the column W contains the pitch in inches.

Column X contains the breadth of the teeth also in inches.

Column Y is formed upon the supposition, that the teeth of all the pitches were of the same length, and contains the strength and durability of the teeth valued in horses' power.

Column Z contains the strength and durability of the teeth, also valued in horses' power, upon the supposition, that the lengths of the teeth are in the same pro-

portion to one another, as the pitches ; and that the strength (by Prop. II.) is inversely as the length. Having taken a three inch pitch as our standard, the two last columns, Y and Z, exactly coincide for that pitch ; the column Z, decreasing upwards from that point, and increasing downwards in an inverse ratio of the lengths.

It is evident, that the tables upon these principles already laid down, might be greatly extended. But it is hoped, that these will be sufficient for our present purpose.

I. *Table of Pitches.*

90. The velocity of the pitch line, being three feet per second, and the breadth of the teeth nine inches.

W Y Z

Pitch in inches.	Value of strength in horses' power.	Value of strength in horses' power.
4	16.	12.
$3\frac{1}{2}$	12.25	10.5
3	9.	9.
$2\frac{1}{2}$	6.25	7.5
2	4.	6.
$1\frac{1}{2}$	2.25	4.5
1	1	3.

Supposing again, that the pitches were the same as in the first table, and that the breadths were made, in each particular case, just double the pitch, then the horses' power would vary as in the following ta-

N

ble, which is calculated by taking the above table, and by direct proportion, finding the horses equal to each particular breadth, when the breadth is just double the pitch.

II. *Table of Pitches.*

91. The velocity being three feet per second, and the breadth of the teeth double each pitch.

W X Y Z			
Pitch in inches.	Twice the pitch in breadth.	Value of strength in horses' power.	Value of strength in horses' power.
4	8	14.22	10.66
3½	7	9.53	8.17
3	6	6.	6.
2½	5	3.47	4.16
2	4	1.77	2.65
1½	3	.75	1.5
1	2	.22	.66

The strength being directly as the breadth, (by Prop. I.) it is easy from this to find, by the Rule of Three direct, the horses' power equal to any given breadth of these pitches.

The stress being inversely as the velocity, (by Prop. II.) the horses' power equal to any of these pitches, and breadths, may be easily found for any other velocity.

But, perhaps, it will be of advantage to take a different view of the subject, by fixing upon a case in the table of wheels in actual use, and proportioning breadths and pitches from it for various velocities, resistances, &c.

For this purpose we shall select the case H, erected by Messrs. Boulton and Watt, being a pitch of three inches, the teeth 8 inches broad, moving with a velocity of 11 feet per second, and having a resistance valued at the power of 46 horses.

Then by following a similar procedure, with the two foregoing tables, we have the tables III. and IV. proportionate to the case H at a velocity of 11 feet per second.

III. *Table of Pitches*

92. Proportionate to H in the Table of Wheels. The breadth of teeth (8 inches) and velocity (eleven feet per second) being constant.

W Y Z		
Pitch in inches.	Horses' power.	Value of strength in horses' power.
4	81.77	61.33
$3\frac{1}{2}$	62.61	53.66
3	46.	46
$2\frac{1}{2}$	31.94	38.33
2	20.44	30.66
$1\frac{1}{2}$	11.5	23.
1	5.11	15.33

IV. *Table of Pitches,*

93. Proportionate to H, the breadth being in this case double the pitch. The velocity being constant (eleven feet per second), but the breadths double.

W X Y Z

Pitch in inches.	Breadths double the pitch in inches.	Horses' power.	Value of strength in horses' power.
4	8	81.77	61.33
3½	7	54.78	46.95
3	6	34.5	34.5
2½	5	19.95	23.94
2	4	10.22	15.33
1½	3	4.31	8.62
1	2	1.28	3.84

But it may be satisfactory, in order to compare with the tables, first and second, to reduce the velocity to three feet per second.

The two following tables, therefore, are calculated accordingly at that velocity.

V. Table of Pitches,

94. Proportionate to H, at a velocity of three feet per second; the breadth being constantly eight inches.

W Y Z W

Pitch in inches.	Horses' power.	Value of strength in horses' power.
4	22.30	16.72
3½	17.07	14.63
3	12.54	12.54
2½	8.71	10.45
2	5.57	8.36
1½	3.13	6.26
1	1.39	4.17

VI. *Table of Pitches,*

95. Proportionate to H, at a velocity of three feet; the breadths being double each pitch.

W	X	Y	Z
Pitch in inches.	Breadth of teeth in inches.	Horses' power.	Value of strength in horses' power.
4	8	22.30	16.72
$3\frac{1}{2}$	7	14.93	12.79
3	6	9.4	9.4
$2\frac{1}{2}$	5	5.44	6.53
2	4	2.78	4.17
$1\frac{1}{2}$	3	1.17	2.34
1	2	0.34	1.02

From a comparison of the second table with the sixth, it appears, that the rule we have annexed, is at least safe in point of strength; for by that rule, a *three inch*

pitch, six inches broad, is equal to a strain of six horses. Whereas, in the sixth table, the same pitch and velocity, at six inches breadth, has strength valued at nine and nearly a half horses' power.

But when we consider how much more liable, from sand, &c. teeth attached to water wheels are to wear, than those which are properly greased and free from sand, the results correspond as nearly as could be expected.

96. Rule II. So that taking H as a standard, we may conclude, that, *for a pitch of three inches*, with a velocity of three feet per second, every inch of breadth may be valued at one and a half horses' power.

The first rule (Art. 88.) I think therefore may safely be followed for teeth attached to water wheels, and the above conclusion for wheels in all situations where they are properly greased and free from sand.

The conclusions here drawn, will, I think, give teeth sufficiently durable and strong for the work which they may have to perform. This first conclusion gives, perhaps, too great a result: how much may with prudence be deducted from the results of either, those of experience will determine. But to the young millwright, I would advise, of the two extremes, rather to err in making his work too strong. Durability ought not for a moment to be out of sight in the arrangements of wheel work; there are many parts of machines subjected to great stress, but not liable to wear; whereas the teeth of wheels, the moment they begin to act, begin to change from their original form, and to become progressively less strong.

97. In millwork, at present, the breadth of the teeth, as commonly executed by the best masters, seems to be from about twice to thrice the pitch.

It is, perhaps, not easy to determine

what proportion is on the whole the most advantageous for the breadth of teeth. A fine pitch on the one hand, gives a smooth motion, and the teeth will rub less on each other; but an increase of breadth increases in some degree the friction.* The durability, as well as the strength of teeth, is perhaps nearly in direct proportion to their breadth.

98. After sending the foregoing "*Inquiry respecting the Strength of the Teeth of Wheels*" to the press, Mr. John Robertson, engineer, perused a manuscript copy of it, and was so obliging as to communicate to me the substance of what follows. His rule, it will be readily perceived, is founded on the principles laid down in the "*Inquiry*;" but it is more simple, and perhaps more accurate than the mode of approximation which occurred to me. It is however satisfactory to find, that the table formed on his rule,

* See Art. 48 and 49.

(which from his experience he is of opinion cannot be far from the truth,) very nearly coincides in its results with tables fifth and sixth. It may be observed, that he founds his calculations upon *the thickness of the teeth*, which in all cases he supposes a little less than half the pitch, which proportion is very common in practice. When both wheel and pinion, however, are of cast iron, it is evident, that being more liable to wear, the teeth of the pinion ought to be thicker than those of the wheel.

Construction of the following Table.

99. *The thickness of the teeth* in each of the lines, is varied one tenth of an inch. *The breadth of the teeth* is always four times as much as their thickness. *The strength* of the teeth is ascertained by multiplying the square of their thickness into their breadth, taken in inches and tenths, &c. The pitch is found by multiplying the

thickness of the teeth by 2.1. The number that represents the strength of the teeth, will also represent the number of horses' power, at a velocity of about four feet per second. Thus in the table where the *pitch is 3.15 inches, the thickness of the teeth 1.5 inches, and the breadth 6. inches, the strength is valued at 13½ horses' power, with a velocity of four feet per second at the pitch line.*

A Table of Pitches of Wheels.

100. With the *breadth* and *thickness* of the teeth, and the corresponding number of horses' power, moving at the pitch line at the rate of three feet, of four feet, of six feet, and of eight feet per second.

Pitch in inches.	Thickness of teeth in inches.	Breadth of teeth in inches.	Strength of teeth, or number of horses' power at 4 feet per second.	Horses' power at 3 feet per second.	Horses' power at 6 feet per second.	Horses' power at 8 feet per second.
3.99	1.9	7.6	27.43	20.57	41.14	54.85
3.78	1.8	7.2	23.32	17.49	34.98	46.64
3.57	1.7	6.8	19.65	14.73	29.46	39.28
3.36	1.6	6.4	16.38	12.28	24.56	32.74
3.15	1.5	6.	13.5	10.12	20.24	26.98
2.94	1.4	5.6	10.97	8.22	16.44	21.92
2.73	1.3	5.2	8.78	6.58	13.16	17.54
2.52	1.2	4.8	6.91	5.18	10.36	13.81
2.31	1.1	4.4	5.32	3.99	7.98	10.64
2.1	1.0	4.	4.0	3.0	6.0	8.0
1.89	.9	3.6	2.91	2.18	4.36	5.81
1.68	.8	3.2	2.04	1.53	3.06	3.08
1.47	.7	2.8	1.37	1.027	2.04	2.72
1.26	.6	2.4	.86	.64	1.38	1.84
1.05	.5	2.	.5	.375	.75	1.

I HOPE the following letter, respecting the strength, &c. of wheel-work, for which I am indebted to Mr. JAMES CARMICHAEL, millwright, (now of Dundee,) will not be unacceptable to the reader:

101. “SIR—It is a corroboration of the truth of the tables of pitches, that Mr. Roberton’s table coincides very nearly with columns marked Y in your tables; but he seems to have overlooked the propriety of taking the length of the teeth into his calculations. I am, therefore, still of opinion, that the true value is in the columns marked Z in your tables.

“Admitting the truth of the fundamental propositions, and, from a comparison of the tables of pitches, I would propose the following rule, which is on the same principle as that of columns Z in your tables, for calculating the proportionate strength of the teeth of wheels.

Rule.

“ Multiply the breadth of the teeth by the square of the thickness, and divide the product by the length. The quotient will be the proportionate strength in horses’ power, with a velocity of 2·27 feet per second.

“ By that rule I have calculated the following table; and, for the sake of comparison, I have taken three cases from Mr. Roberton’s table, and three from your tables 3d and 5th.

Explanation of the Table.

“ Column 2, contains the thickness of the teeth. The pitch is found by multiplying the thickness by 2·1*; and the length

* That is in order to make the space between the teeth a little wider than the thickness of a tooth. See Art, 99.

is found by multiplying the thickness by 1.2.*

“Column 5, contains the proportionate strength, and also the number of horses’ power (proportionate to the case H, see page 179), which the teeth is equal to, with a velocity of 2.27 feet per second.

1	2	3	4	5	6	7	8
Pitch in inches.	Thick-ness of teeth in inches.	Breadth of teeth in inches.	Length of teeth in inches.	Strength of teeth, or num-ber of horses’ power, at 2.27 feet per second.	Horses’ power at three feet per second.	Horses’ power at six feet per second.	Horses’ power at 11 feet per second.
3.9	1.9	7.6	2.28	11.73	15.46	30.92	56.84
2.9	1.4	5.6	1.68	6.53	8.63	17.26	31.64
2.1	1.	4.	1.2	3.33	4.4	8.8	16.1
4.	1.904	8.	2.285	12.698	16.78	33.56	61.52
3.	1.428	8.	1.714	9.523	12.58	25.16	46.
1½.	.714	8.	.857	4.752	6.27	12.54	23.02

Remarks.

“1st, The last three cases in the table are taken from the tables 3d and 5th, and the results coincide so well with the co-

* Respecting the length of teeth, see Art. 103.

lumns marked Z, that I presume the rule is just.

“ 2dly, The first three cases are from Mr. Robertson’s table; the second case is very near the same as in Mr. R’s table; but the first is considerably less, and the third considerably more. Hence I infer that Mr. R. has taken his data from a pitch about three inches.

“ 3dly, If any two wheels have the length and thickness of their teeth in the same proportion to their respective pitches, the breadth of the teeth and the velocity being the same, *the strength will be directly as the pitches*. The truth of this is deduced from the columns marked Z in tables 1st, 3d, and 5th.

“ I am,

“ SIR,

“ Your most obedient servant,

“ JAMES CARMICHAEL.”

Glasgow,
DECEMBER, 1808.

102. TABLE OF PITCHES OF WHEEL- WORK,

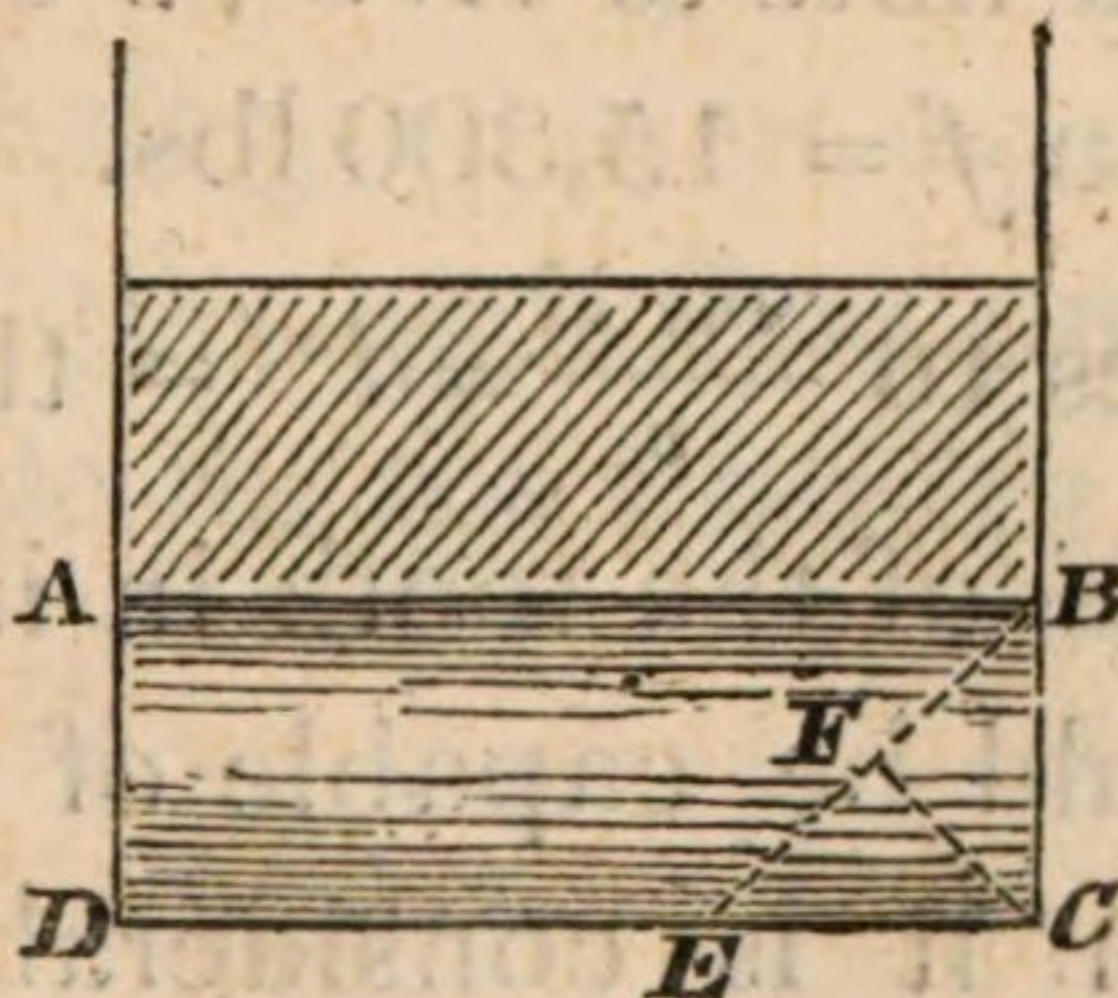
With the *breadth* and *thickness* of the teeth, and the corresponding strength in horses' power, calculated by Mr. CAR-MICHAEL'S rule.

Pitch in inches.	Thick-ness of teeth in inches.	Breadth of teeth in inches.	Length of teeth in inches.	Strength of teeth, in horses' power at 2·27 feet per second.	Horses' power, at three feet per second.	Horses' power, at six feet per second.	Horses' power at eleven feet per second.
3·99	1·9	7·6	2·28	12·03	15·90	31·80	58·30
3·78	1·8	7·2	2·16	10·80	14·27	28·54	52·32
3·57	1·7	6·8	2·04	9·63	12·72	25·54	46·68
3·36	1·6	6·4	1·92	8·53	11·27	22·54	41·32
3·15	1·5	6·0	1·80	7·50	9·91	19·82	36·33
2·94	1·4	5·6	1·68	6·53	8·63	17·26	31·64
2·73	1·3	5·2	1·56	5·63	7·44	14·88	27·28
2·52	1·2	4·8	1·44	4·80	6·34	12·68	23·24
2·31	1·1	4·4	1·32	4·03	5·32	10·64	19·54
2·10	1·0	4·0	1·20	3·33	4·40	8·81	16·15
1·89	0·9	3·6	1·08	2·70	3·57	7·14	13·09
1·68	0·8	3·2	0·96	2·13	2·81	5·62	10·33
1·47	0·7	2·8	0·84	1·63	2·15	4·30	7·88
1·26	0·6	2·4	0·72	1·20	1·59	3·18	5·83
1·05	0·5	2·0	0·60	0·83	1·10	2·20	4·03

[A. 102.] It is not perhaps quite so difficult, as our author imagined, to determine, from first principles, the strength proper for teeth of wheels, and such a method must always be preferred to empirical rules. I shall here shew how to apply those principles which will give the reader an opportunity of comparing the two methods.

In the first place let us consider under what circumstances the strain on a tooth will be the greatest possible.

Let A B C D be the side of a tooth, then it will be evident, that the strain will be



greatest, when the stress is thrown upon

one corner of the tooth, as at C, whether it be from irregular action or from any substance getting between the teeth.

In such a case, it may be shown by the rules of *maxima* and *minima*, that E C being equal to C B, the strain will be greatest in the line E B; and, in the case of fracture, it would take place according to that line.

Since the thickness of a tooth is not regular, we shall have a result, sufficiently near for this purpose, if we express the relation between the stress and strain by $\frac{740 H}{v} = \frac{.7854 f^2 (F C) d^2}{8 (F C)}$. (*Essay on Cast Iron*, Art. 81. and note to Art. 78 of this Essay,) which, when $f = 15,300$ lbs. on a square inch, reduces to $\frac{.247 H}{v} = d^2 =$ the square of the thickness of the tooth in inches. But, a tooth should be capable of resisting this stress, when it is considerably worn by friction; and an allowance fully equal to that which ought to take place before re-

newing the wearing parts of the machine, will be one third of the thickness of a tooth. Now to allow of this degree of wear, and remain equal to the stress, it may be easily shown that the tooth should be capable of resisting $2\frac{1}{4}$ times the power at the first; therefore $\frac{.556 H}{v} = d^2$; or $\frac{3}{4} \sqrt{\frac{H}{v}} = d$. Where H is the number of horses which are equal to the power of the first mover; v the velocity of the pitch line of the wheel in feet per second, and d the thickness of a tooth in inches.

This investigation furnishes an easy practical rule for the thickness of teeth; and, consequently, for the pitch of wheels and pinions; I shall give it in words at length, with an example, and then proceed to determine a rule for the breadth of teeth.

*Rule for the thickness of Cast-iron Teeth
for Wheels.*

[B. 102.] Find the number of horses which are equivalent to the power of the first mover of the train of machinery, and divide that number by the velocity, in feet per second, of the pitch line of the pinion or wheel ; extract the square root of the quotient, and three fourths of this root will be the least thickness of the tooth for the wheel or pinion, in inches.

*Rule for the least quantity of Pitch for a
Wheel or Pinion with Teeth of Cast Iron.*

[C. 102.] If the thickness of the teeth of the pinion be intended to be the same as those of the wheel, multiply the thickness above determined by 2.1, the product will be the pitch required.

But we may observe, that if a pinion makes three turns, for example, while the

wheel makes one, the teeth of the pinion will be worn three times the quantity of those of the wheel. Hence, to provide against such excess of wear, if the pinion makes n revolutions, while the wheel makes one, the pitch should be $\frac{5.3+n}{3} d$ inches, and the thickness of the teeth of the pinion $\frac{2+n}{3} d$ inches, when d is the thickness of the teeth of the wheel.

EXAMPLE.

[D. 102.] Let the force of the first mover be equivalent to ten horses, and the velocity of the pitch line three feet per second. Dividing 10 by 3, we have $3\frac{1}{3}$; and the square root of $3\frac{1}{3}$ (by the table of powers, Art. 373.) is 1.83 nearly; and $\frac{3}{4} \times 1.83 = 1.45$ inches for the thickness of the teeth of the wheel.

Again, suppose the pinion to turn twice to the wheel once, then $n = 2$, and $\frac{5.3+2}{3} \times 1.45 = 3.5$ inches the pitch.

And $\frac{2+n}{3} \times 1.45 = 1.93$ inches, the thickness of the teeth of the pinion.

Thickness of Wooden Teeth.

[E. 102.] The kind of wood employed for teeth, is usually about one fourth of the strength of cast iron, and since the thickness of the teeth should vary inversely as the square root of the power of the material, the square root of $\frac{1}{4}$ being $\frac{1}{2}$, wooden teeth should be twice the thickness of cast iron ones. The pitch of course will be greater in the same proportion.

To determine the breadth of Cast-iron Teeth.

[F. 102.] That case where a beam is fixed at one end, and the load acts at the other, applies to the teeth of wheels, in their general state of action, and the stress is $\frac{740 H}{v}$. (note to Art. 78.) Hence, when l = the length, and b = the breadth of a tooth, $\frac{740 H l}{12 v} = 212 b d^2$. (*Essay on Cast*

Iron, Art. 116,) and to allow one third of the thickness of the tooth for wear, the equation becomes $\frac{2\frac{1}{4} \times 740 H l}{12 v} = 212 b d^2$. But we have already seen, that $d^2 = \frac{.556 H}{v}$; therefore $\frac{2\frac{1}{4} \times 740 \times l}{12} = .556 b \times 212$, or $1.2 l = b$.

This calculation informs us what breadth is essential for strength; that is, the breadth should never be less than 1.2 multiplied by the length of the tooth; but, it may be proved that the durability is nearly in direct proportion to the breadth, and inversely as the pressure. Some of the maxims of our author as far as regards the breadths of teeth agree well with the theory of durability; but the conclusions respecting strength, and the limits of pitch are not so much to be relied upon; indeed the facts drawn from practical construction are not so well adapted for the latter object.

[G. 102.] If we suppose that teeth 6 inches in breadth are sufficiently durable

for a power equivalent to 10 horses, when the pitch line moves at the rate of 3 feet per second ; and the Table of Wheels in p. 169, Art. 86. seems to indicate that this supposition is near the truth ; therefore $\frac{10}{3} : 6 :: \frac{H}{v} : b = \frac{3 \times 6 \times H}{10v} = \frac{1.8H}{v}$. That is, multiply the horses' power by 1.8, and divide by the velocity of the pitch line in feet per second, the quotient will be the breadth in inches.

EXAMPLE. Taking the case H in the table Art. 86, we have $\frac{1.8H}{v} = \frac{1.8 \times 46}{11} = 7.53$ inches : Messrs. Boulton and Watt in this case made the breadth 8 inches.

In the case K, by the same makers, $\frac{1.8H}{v} = \frac{1.8 \times 14}{6.65} = 3.8$ inches. The breadth actually employed was 5 inches. Hence it appears, that in a considerable range of power and velocity, our formula gives results below those actually employed, but the breadth assigned by the table calculated

by Mr. Roberton, Art. 100, is always vastly below mine in the greater powers ; indeed it is manifestly erroneous in the breadths, for where the moving power is doubled, the breadth is increased only one third. In the thickness of teeth, it nearly agrees with my rule.

Breadth of Wooden Teeth.

[H. 102.] For wooden teeth we may take as the basis of a practical rule, the case N in the table, Art. 86 ; which, expressed in the nearest whole number, is $\frac{5H}{v} = b$ in inches.

EXAMPLE.

In the case O ; $H=4$ horses, and $v=4.8$ feet per second ; therefore, $\frac{5H}{v} = \frac{5 \times 4}{4.8} = 4.17$ inches the breadth ; the actual breadth used was $4\frac{3}{4}$ inches.

Of the Strength of Staves for Trundles.

[I. 102.] This is a subject our author has not touched upon, but I consider it necessary to examine it, because trundles seem capable of improvement, and they have some advantages which toothed pinions have not.

If the length of a stave in feet be l , its diameter in inches d , and the stress upon it $\frac{740 H}{v}$; which is supposed to act at the weakest part of the stave; that is, in the middle of its length; then, by the rule for the strength of cylinders, (*Essay on Cast Iron*, Art. 129,) $\frac{740 H l}{500 v} = d^3$. Now, if it be made to resist $2\frac{1}{4}$ times the power when first made, in order to allow for wear; we shall have $\frac{2\frac{1}{4} \times 740 H l}{500 v} = d^3$, or $(\frac{3.33 H l}{v})^{\frac{1}{3}} = d$; or with sufficient accuracy, $2 (\frac{H l}{v})^{\frac{1}{3}} = d$.

EXAMPLE.

Let the power of the first mover be equal to 10 horses, the velocity of the pitch line 3 feet per second, and the length of the stave $\cdot 9$ of a foot; then $2 \left(\frac{Hl}{v} \right)^{\frac{1}{3}} = 2 \left(\frac{10 \times \cdot 9}{3} \right)^{\frac{1}{3}} = 2 \times 1.442 = 2.884$ inches the diameter of the stave. (Ed.)

103. As it may be of use to millwrights, I take the liberty of inserting the following tables from a respectable periodical publication.*

* This very useful table was printed in a small pamphlet, price one shilling, in 1803, but is at present out of print. When the number of teeth is less than 10, look for the radius of double the number of teeth, and half of it will be the radius required. When the number of teeth exceeds 300, look for the radius of half the number of teeth, the double of which will be the one required. (Ed.)

Table of the Radii of Wheels, from Ten to Three Hundred Teeth, the Pitch being two Inches. By Mr. B. Donkin, Civil Engineer, London.*

No. of teeth.	Radius in inches.	No.	Radius.	No.	Radius.
10	3.236	34	10.838	58	18.471
11	3.549	35	11.156	59	18.789
12	3.864	36	11.474	60	19.107
13	4.179	37	11.792	61	19.425
14	4.494	38	12.110	62	19.744
15	4.810	39	12.428	63	20.062
16	5.126	40	12.746	64	20.380
17	5.442	41	13.064	65	20.698
18	5.759	42	13.382	66	21.016
19	6.076	43	13.700	67	21.335
20	6.392	44	14.018	68	21.653
21	6.710	45	14.336	69	21.971
22	7.027	46	14.654	70	22.289
23	7.344	47	14.972	71	22.607
24	7.661	48	15.290	72	22.926
25	7.979	49	15.608	73	23.244
26	8.296	50	15.926	74	23.562
27	8.614	51	16.244	75	23.880
28	8.931	52	16.562	76	24.198
29	9.249	53	16.880	77	24.517
30	9.567	54	17.198	78	24.835
31	9.885	55	17.517	79	25.153
32	10.202	56	17.835	80	25.471
33	10.520	57	18.153	81	25.790

* By the Pitch is understood the distance between the centres of two contiguous teeth; and by the radius is understood the distance between the centre of the wheel and the centre of each tooth. For any other pitch, say, as two inches is to the radius in the table, so is the given pitch to the radius required.

No.	Radius.		No.	Radius.		No.	Radius.
82	26.108		118	37.565		154	49.023
83	26.426		119	37.883		155	49.341
84	26.741		120	38.202		156	49.660
85	27.063		121	38.520		157	49.978
86	27.381		122	38.838		158	50.296
87	27.699		123	39.156		159	50.615
88	28.017		124	39.475		160	50.933
89	28.336		125	39.793		161	51.251
90	28.654		126	40.111		162	51.569
91	28.972		127	40.429		163	51.888
92	29.290		128	40.748		164	52.206
93	29.608		129	41.066		165	52.524
94	29.927		130	41.384		166	52.843
95	30.245		131	41.703		167	53.161
96	30.563		132	42.021		168	53.479
97	30.881		133	42.339		169	53.798
98	31.200		134	42.657		170	54.116
99	31.518		135	42.976		171	54.434
100	31.836		136	43.294		172	54.752
101	32.155		137	43.612		173	55.071
102	32.473		138	43.931		174	55.389
103	32.791		139	44.249		175	55.707
104	33.109		140	44.567		176	56.026
105	33.427		141	44.885		177	56.344
106	33.746		142	45.204		178	56.662
107	34.064		143	45.522		179	56.980
108	34.382		144	45.840		180	57.299
109	34.700		145	46.158		181	57.617
110	35.018		146	46.477		182	57.935
111	35.337		147	46.795		183	58.253
112	35.655		148	47.113		184	58.572
113	35.974		149	47.432		185	58.890
114	36.292		150	47.750		186	59.209
115	36.611		151	48.068		187	59.527
116	36.929		152	48.387		188	59.845
117	37.247		153	48.705		189	60.163

No.	Radius.		No.	Radius.		No.	Radius.
190	60.482		227	72.258		264	84.036
191	60.800		228	72.577		265	84.354
192	61.118		229	72.895		266	84.673
193	61.436		230	73.214		267	84.991
194	61.755		231	73.532		268	85.309
195	62.073		232	73.850		269	85.627
196	62.392		233	74.168		270	85.946
197	62.710		234	74.487		271	86.264
198	63.028		235	74.805		272	86.582
199	63.346		236	75.123		273	86.900
200	63.665		237	75.441		274	87.219
201	63.983		238	75.760		275	87.537
202	64.301		239	76.078		276	87.855
203	64.620		240	76.397		277	88.174
204	64.938		241	76.715		278	88.492
205	65.256		242	77.033		279	88.810
206	65.574		243	77.351		280	89.129
207	65.893		244	77.670		281	89.447
208	66.211		245	77.988		282	89.765
209	66.529		246	78.306		283	90.084
210	66.848		247	78.625		284	90.402
211	67.166		248	78.943		285	90.720
212	67.484		249	79.261		286	91.038
213	67.803		250	79.580		287	91.357
214	68.121		251	79.898		288	91.675
215	68.439		252	80.216		289	91.993
216	68.757		253	80.534		290	92.312
217	69.075		254	80.853		291	92.630
218	69.394		255	81.171		292	92.948
219	69.712		256	81.489		293	93.267
220	70.031		257	81.808		294	93.585
221	70.349		258	82.126		295	93.903
222	70.667		259	82.444		296	94.222
223	70.985		260	82.763		297	94.540
224	71.304		261	83.081		298	94.858
225	71.622		262	83.399		299	95.177
226	71.941		263	83.717		300	95.495

*Of Arranging the Numbers of Wheel
Work.*

[A. 103.] In a machine, the velocity of the impelled point should be to that of the working point in the ratio which is adapted to the maximum effect of the moving power on the one part, and the best working effect on the other part. Any other arrangement of the relative motions of the parts of a machine must clearly be attended with a loss of power, or the work will not be done properly. But when the best working velocity is known, and also that which enables the first mover to produce the greatest effect; the proper arrangement of the numbers of the teeth of the wheels and pinions is a very simple operation. The subject has been treated of for particular machines, by several writers; but since it has been chiefly under a somewhat erroneous view of the real nature of the maximum effect of machines, it will be perhaps of use to give a general

formula, and a few particular examples, to save the trouble of reference, and render the work somewhat more complete.*

It will be an advantage to advertise the young mechanic of one or two essential particulars, before proceeding to the principal object.

[B. 103.] In the first place, When the wheels drive the pinions, the number of teeth in any one pinion, should not be less than 8; but rather let there be 11 or 12 if it can be done conveniently. And in the particular form of teeth described in Art. 26, the number of teeth in a pinion should not be less than 10; but it would

* The methods of adjusting the numbers of wheel work, so that the contemporary revolutions may be always in a given ratio, is a distinct branch of this subject, chiefly useful in clock and watch-work, planetary machines, and the like; and since our plan does not include the construction of such machines, the reader desirous of such information, may consult Camus on the Teeth of Wheels.

be better to have 13 or 14. (See Art. A 28, to D 28.)

[C. 103.] Secondly, When the pinions drive the wheels, the number of teeth on a pinion may be less ; but it will not in any case be desirable to have fewer than 6 teeth on a pinion ; and give the preference to 8 or 9, where it can be done with convenience.

[D. 103.] Thirdly, The number of teeth in a wheel should be prime to the number of teeth in its pinion ; that is, the number representing the teeth in the wheel should not be divisible by the number of teeth in the pinion without a remainder. And as the numbers of pinions will in general be first settled, it will be an advantage to take a prime number for each pinion, as 7, 11, 13, 17, 19, 23, &c. because such numbers are seldomer factors than others. But when it happens that a prime number can be directly fixed upon for the wheel, any whole number which approaches near

to the required ratio will answer for the pinion; as minute accuracy is not required. A prime number for the wheel, or one which is not divisible by the number of the pinion, is esteemed the best, because the same teeth will not always come together, and the wear will be more uniform.

[E. 103.] Fourthly, If it be desired that a given increase or decrease of velocity should be communicated with the least quantity of wheel work, it has been shown that the number of teeth on each pinion should be to the number on its wheel, as 1 : 3.59 (Dr. Young's Nat. Phil. Vol. II. art. 366.) But, on account of the space required for several wheels, and the expence of them, it will often be necessary to have 5 or 6 times the number of teeth on the wheel that there is on the pinion. The ratio of 1 : 6 should however not be exceeded, unless there be some other important reason for a higher ratio.

[F. 103.] *Of calculating the Numbers*

for Wheel Work. Let n be the number of revolutions per minute for the first axis, to which the moving power gives motion; and N the number of revolutions per minute of the last axis, where the resistance or working point is. Then, $n : N :: 1 : \frac{N}{n}$, which is the ratio the velocity is to be increased or diminished.

If this ratio should not exceed $1 : 6$ a single wheel and pinion will be sufficient; but when it exceeds that ratio, more will be necessary.

When each of the pinions has the same number of teeth, and each wheel the same number; then, the ratio of the number on a pinion, will be to the number on a wheel as $1 : \frac{N^{\frac{1}{a}}}{n^{\frac{1}{a}}}$; or as $1 : \left(\frac{N}{n}\right)^{\frac{1}{a}}$. (1).

The number of pinions being a .

But it is sometimes necessary to adapt the trains of machinery to produce differ-

ent velocities at the working points, and it is on this account often desirable to vary the size of the wheels. Then, the ratio $1 : \frac{N}{n}$ must be decomposed into factors suitable to the nature of the work to be done; if those factors be any numbers a, b, c , &c. the ratio will be $1 : a \times b \times c$, &c. $:\frac{N}{n}$. (2.)

The first mover should be as near as possible to the resistance. But, when it is absolutely necessary to perform operations at a considerable distance from the first mover, the velocity of the communicating shafts should be brought up, as near as possible to the first mover, to that which is most advantageous for the different species of work.

But the velocities of the parts of machines are often given in feet per second; let v be the velocity in feet per second; then $60v$ is the feet described in a minute. Also let $d \times 3.1416$ be the circumference

which moves with the velocity v ; then

$\frac{60 v}{3.1416 d} = \frac{19.09 v}{d}$ is the revolutions the axis makes in a minute.

When the first mover acts with a velocity v at the distance $\frac{1}{2} d$ from the axis;

then $\frac{19.09 v}{d} = n$; and the ratio the teeth on the pinions should be to those on the wheels as $1 : \left(\frac{N d}{19.09 v} \right)^{\frac{1}{a}}$. (3.)

And, when the velocity V of the working point is also given, then $\frac{19.09 V}{D} = N$; and the ratio of the teeth on the pinions should be to those on the wheels as $1 : \left(\frac{V d}{v D} \right)^{\frac{1}{a}}$. (4.)

If the velocity is to be decreased, then it will be as the wheels are to the pinions, instead of the pinions to the wheels. The use of these proportions will be best illustrated by examples.

[G. 103.] EXAMPLE I. Let it be required to calculate the numbers for a

corn mill moved by an overshot water wheel.

This case comes under Proportion (4). where a , the number of pinions, will never exceed 2, and not often more than 1. And in order that the grain may not be too much heated in grinding, the velocity of the circumference of the millstone, should not be greater than 23 feet per second, hence $V=23$; and D will be the diameter of the millstone in feet.

Also, in an overshot wheel $v = 2.67 \sqrt{h}$ (Art. 328. B.) and $d = 1.108 h$, (Art. 328, A.) where h is the whole fall in feet. Hence 1 :

$\left(\frac{V d}{v D}\right)^{\frac{1}{a}}$, becomes $1 : \left(\frac{23 \times 1.108 h}{2.67 \sqrt{h} \times D}\right)^{\frac{1}{2}}$. Or as

$1 : \left(\frac{9.54 \sqrt{h}}{D}\right)^{\frac{1}{2}}$ when there are two pinions ;

and simply as $1 : \frac{9.54 \sqrt{h}}{D}$, when there is but one pinion. Suppose the diameter of the stone to be 5 feet, and the fall 16 feet, then

$\sqrt{h} = 4$, and $D = 5$, and $1 : \frac{9.54 \times 4}{5}$, or as

1 : 7.632. Now it will be better, in this case, to have two pinions, since the ratio is greater than 1 : 6 ; therefore as 1 : 7.632¹ or as 1 : 2.763, so is the teeth in each pinion to the teeth in its wheel. And making the prime number 11 the number of teeth of each pinion, we shall have $11 \times 2.763 = 30$, the nearest whole number for the teeth of each wheel. The thickness of the teeth being found by the rules for that purpose, (see Art. B. 102.) the radius of the wheels and pinions will be found by the table of radii, (Art. 103.)

Again, let the fall be 4.84 feet, the diameter of the millstone being 5 feet as before ;

then, $\sqrt{h} = 2.2$, and $1 : \frac{9.54 \sqrt{h}}{D}$ becomes as

1 : 4.1976. Here one pinion or trundle will be best ; and making the teeth on the pinion 11 those on the wheel will be $11 \times 4.1976 = 46$ in the nearest whole number.

The young millwright will find it useful

to calculate a table by these rules, which might be extended to undershot wheels, and exhibit at one view the whole construction of mills.

[H. 103.] EXAMPLE II. Let it be required to arrange the numbers for a machine for raising water, where the pumps are to make N strokes per minute, the velocity of the moving power being v feet per second, and the diameter of the circle described by the power d feet.

This case is an example of the use of Proportion (3.) or $1 : \left(\frac{Nd}{19.09v} \right)^{\frac{1}{a}}$. Now let the moving power be a horse, where v is $2\frac{1}{2}$ feet per second, (see p. 167.) and d the diameter of his track, 30 feet; the pump to make 20 strokes per minute, or $N = 20$, then the ratio is $1 : \frac{20 \times 30}{19.09 \times 2\frac{1}{2}}$, or as $1 : 12.6$ nearly. Therefore if the pinion have 13 teeth, the wheel should have $13 \times 12.6 = 164$ teeth in the nearest whole numbers.

But I should prefer making two pinions, and then the ratio will be $1 : \sqrt{12.6}$ or as $1 : 3.549$ nearly, and each pinion having 11 teeth, the wheels should each have $11 \times 3.549 = 39$ teeth.

These examples will perhaps be sufficient to explain the mode of calculation; and when once it is understood, it will very easily be applied to other cases; and will be found somewhat more convenient than the methods usually followed. (Ed).

PRACTICAL OBSERVATIONS WITH REGARD TO MAKING OF PATTERNS OF CAST IRON WHEELS.

104. Having determined the pitch of the wheel strong enough for the purpose to which it is to be applied, the thickness of the tooth serves to regulate the proportionate strength of the other parts.

A very respectable millwright informs

me, that he has for a considerable time adopted the following rule for determining the length of the teeth of wheels, the practical efficacy of which he has found quite satisfactory.

Rule—*Make the length of the teeth equal to the pitch, deducting freedom,* (by the freedom is meant the distance at the top of one tooth, and the root of another measured at the line of centres,) in other words, the distance from root to root of the teeth, at the line of teeth when the wheels are in action, exactly equal to the pitch.

For example—he makes the teeth of two inches pitch, 1 inch and $\frac{1}{8}$ in length, which is allowing $\frac{1}{8}$ of freedom.

Another respectable millwright, who has had much experience, particularly in mills moved by horses, has for a considerable time past, made the teeth of his wheels in length only one half of the pitch, and

works them as deep as possible, without the point touching the bottoms. Before he fell on this expedient, he found the teeth exceedingly liable to be broken from any sudden motion of the horses.*

Indeed, upon reflection, it will be found there is no occasion for more freedom, than that the point of the tooth of the one wheel, shall just clear the ring of the other; more than this must only serve to weaken the teeth. The mode of geering, however, above alluded to, is more necessary in horse mills than where the moving power is steady and regular.

Hutton (on Clock-work) recommends making the distance of the pitch line $\frac{3}{4}$ of what we call the thickness of the tooth. Thus, suppose the rule applied to a two inch pitch and that the tooth and space were exactly equal, then the tooth would

* Respecting the length of teeth, see Art. A. 25, A. 32, and 36. (Ed.)

project $\frac{1}{4}$ of an inch beyond the pitch line, and its root would be as far within the pitch line, as to receive freely the tooth intended to act on it; suppose it also $\frac{3}{4}$, then the tooth would be $1\frac{1}{2}$ inch long, besides the freedom, which, making as above, $\frac{1}{16}$, the tooth would be in all $1\frac{1}{16}$ inch long.

105. But it is to be remarked, that the millwright, in making his pattern for a cast-iron wheel, has to attend to a circumstance arising from the nature of that material. The pattern must not only be of such a form as to be sufficiently strong, calculating by the bulk of the parts, but also proportioned, so that when the fluid metal is poured in the mould, it may cool in every part nearly at the same time.

When due attention is not paid to this circumstance, as the metal is cooling, if it contract faster in one part than in another, it will be apt to break somewhere, just as a drinking glass is broken by suddenly cooling or heating in any particular part

of it. In all patterns for cast iron, about $\frac{1}{8}$ of an inch to the foot, should be allowed for the contraction of the metal in cooling.

Attention must also be paid to taper the several parts, so that they may rise freely without injuring the mould, when the founder is drawing them out of the sand. A little observation of the operations of a common foundry, will better instruct on this part of the subject than many words. We may observe, however, that about $\frac{1}{16}$ of an inch, in a depth of 6 inches, is commonly a sufficient taper.

106. Attending to those circumstances, I beg leave to offer the following proportions as having been found to answer in practice.

Make the thickness of the ring ae equal to the thickness of the tooth ac near its root. When the ring is made thinner than the root of the tooth, the ring commonly

gives way to a strain, which would not break the tooth.

Make the arm, at the part where it proceeds from the ring, of the same breadth

FIG. 1.

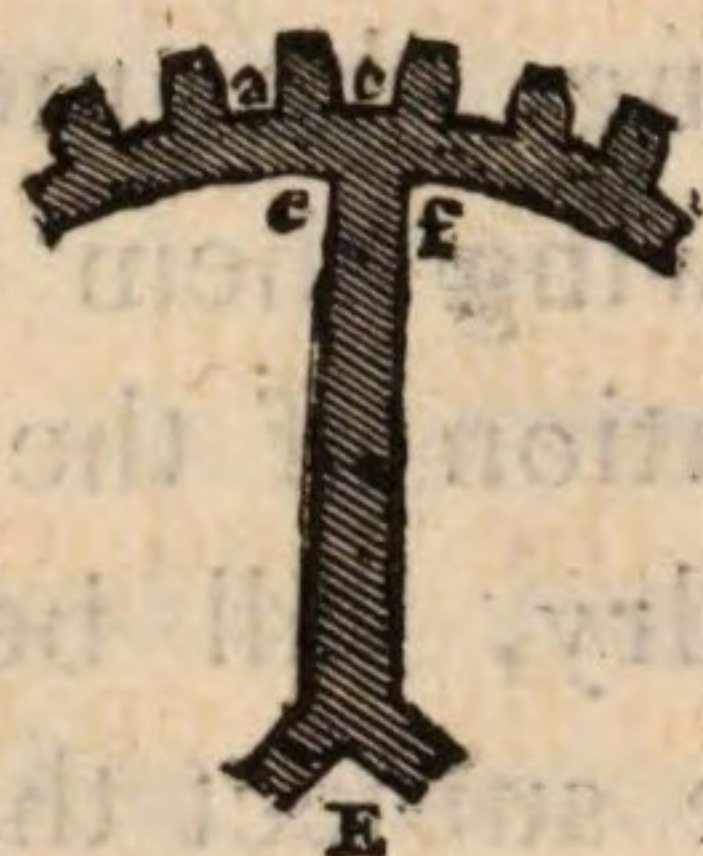


FIG. 2.

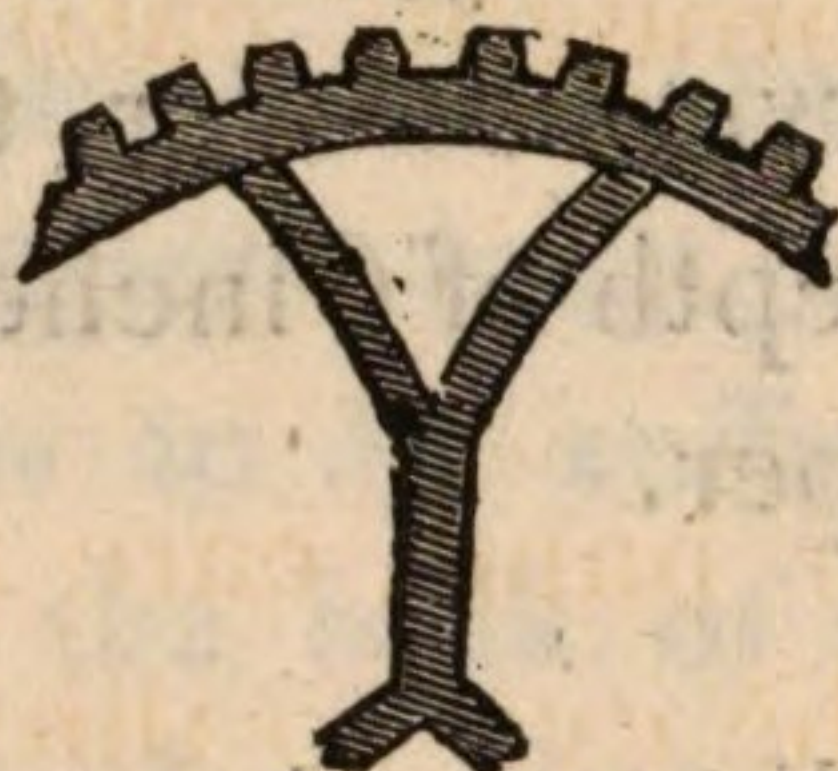


FIG. 3.

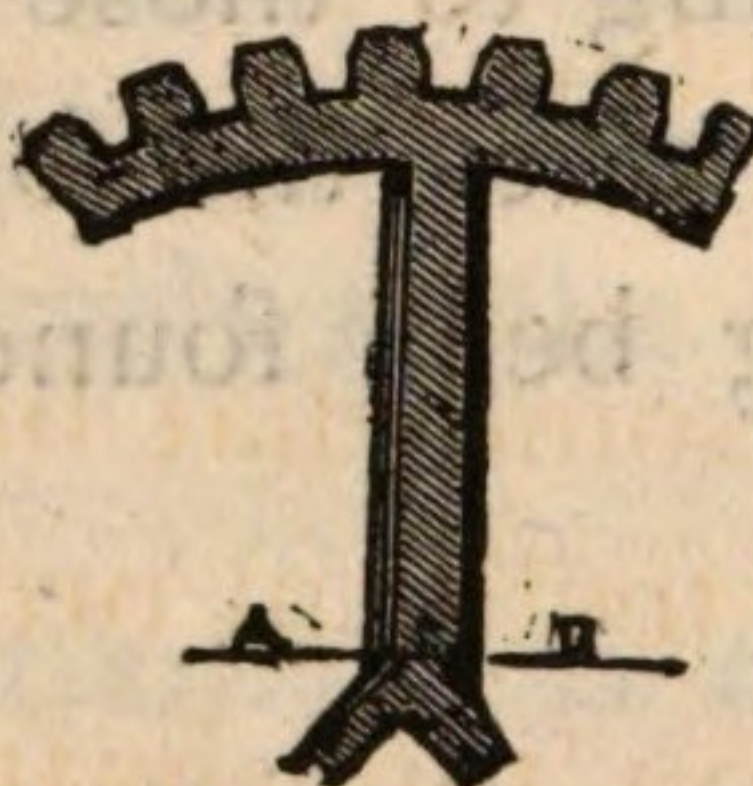


FIG. 4.



and thickness as the ring; and at the junction *e f*, let it be so formed as to take off any acute angle which would be apt to break off in sand.

The arms should become larger as they approach the centre of the wheel, (see Emerson, Prop. 119, Rule 8,) and the eye *E*, should be sufficiently strong to resist the driving of the wedges, by means of which it is to be fixed on the shaft. This cannot be brought easily to calculation.

On the other hand, care must be taken not to make the eye so thick as to endanger unequal cooling.

It should be somewhat broader than the breadth of the teeth, in order that it may be the firmer on the shaft: this breadth must be greater in proportion as the wheel is large.

When the ring *a e* is about an inch

thick, it is common to make the eye about an inch and quarter thickness, and about one-fifth broader than the ring, when the wheel is about four feet diameter.

Small wheels have generally but four arms, but it being improper to have a great space of the ring unsupported, the number of arms should be increased in large wheels.

In order to strengthen the arms with little increase of metal, it is not unusual to make them feathered, which is done by adding a thin plate to the metal at right angles to the arm, as represented by figure third. Fig. 4 is a section of Fig. 3, at A B.

The same rules apply to bevelled wheels; of the practical mode of laying down the working drawings of which we have already spoken. But it is proper to observe, that the eye of a bevelled wheel, should be placed more on that side which

is furthest from the centre of the ideal cone of which the wheel forms a part.

107. When wheels are beyond a certain size, it becomes necessary to have patterns sometimes made for them, cast in parts, which are afterwards united by means of bolts.

A very good mode to prevent the bad effects of unequal contraction, is to have the arms curved, as in the second figure; the curved parts are commonly of the same radius as the wheel, and spring from the half length of the arms.

Materials of Patterns.

108. The patterns should be made of well-seasoned wood. The most proper is clean mahogany, but that being now very expensive, white deal is most commonly

used. Beech is very often used for the teeth, and being a close grained wood, it may be made very smooth.

It is almost superfluous to say, that the workmanship of wheel patterns should be such as to produce great accuracy, and a smooth surface, the former being essential to the good movement of the wheels, and the latter to make the patterns produce a good clean impression in the sand.

I am aware, that it is a common practice, in many places, to make teeth very large in the pattern, and after fixing the wheels on their shafts, to chip and file the teeth to the proper size.

I doubt, however, whether this practice be really advantageous; for besides the great time which it occupies thus to dress the iron teeth, and the consequent expense, there is the loss of the outer skin (if I may use the expression) of the cast iron, which

is by far its most smooth and durable part. In cotton mills, therefore, this method is now but seldom practised.*

* Messrs. Peel, Williams, and Co. have, after great time, trouble, and expense, made and arranged a very great number of patterns of wheels, so as to suit almost every case that can in practice occur. They have published a complete list of them, which they intend inserting also in the "Repertory of Arts." In my opinion, what they have done is a material national benefit; their expense, I am informed, for patterns, has not been less than four thousand pounds. There is, however, every reason to think, that it will be an excellent thing ultimately for themselves, as well as of great practical utility to the public.

APPENDIX, No. II.

TO ESSAY ON TEETH OF WHEELS.

ON THE USE OF CHARTS, AND SOME FURTHER
EXPLANATION OF THE CONSTRUCTION OF THE
TABLES OF PITCHES OF WHEEL-WORK.

109. WHEN quantities of any kind, such as time, space, money, &c. expressed in numbers, are mentioned, it often requires a painful exertion of the mind to recollect and compare them. Hence the utility of bringing them into one view in tables. But there is another mode of comparing quantities not so generally practised, though, in many cases, much more easy and satisfactory to the mind. I allude to charts, in which, instead of using figures, as in tables, the quantities are geometrically represented. This is done by dividing the sides of a square or rectangle into

equal parts, and drawing parallel lines at right angles from the divisions. The quantities are pointed off at certain intersections of these scales.

110. When the quantities increase or decrease in arithmetical proportion, as 1, 2, 3, 4, &c. that proportion will be represented by a straight line, which will pass through these points.

111. But supposing the quantities to increase in geometrical proportion, as 1, 4, 9, 16, &c. the line passing through the points of intersection will form a curve.

These two cases will be best explained by examples.

112. *First*, Suppose the value of any thing to increase as its weight,—the scale on the one side of the square will then represent the value, and that on another the weight.

Let A C Fig. 1, Plate I. represent weight (say ounces,) and A D value (say shillings.)

Now, suppose we mark the price of four ounces, it is done by placing a dot opposite to four, on the line of value, and opposite to four on the line of weight, at the intersections of the perpendiculars from these points, which intersection is marked by *b d* on the figure. In the same manner, we may mark the value of 1, 2, 3, 5, 6 ounces. These points are marked *a b c e f*, and the straight line *A B* passes through them all, and shews the regular progress of the proportion.

It is of no consequence whether the divisions on the line *A C* be greater or less than those on *A D*, provided the lines be divided into equal parts.

113. *Second*, Suppose an accelerating motion, such as that of a falling body, is to be laid down on a chart,—this motion increases as the squares of the times; that is, the body falls a certain distance in the first second of time, four times that distance in the next second, and nine times in the third second, &c.

These points are accordingly marked in Figure 2, by *a* opposite to 1 on both scales, by *b* opposite 2 on the scale of time, A D and 4 on that of motion A C; by *c*, opposite 3, on A D, and 9 on A C, &c. the line A B passing through these points forms a curve.

114. When the proportion of any kind is regular, the curve has a regular easy sweep; if otherwise, the curve will undulate, or have irregular windings. Hence, it is a good mode of proving many kinds of tables, to lay down the quantities thus geometrically; for if there be any material error, when the proportion ought to be regular, an elbow will appear in the line A B.

115. Much calculation, too, may often be saved, for when a few of the principle points at some distance from each other are obtained in the curve, the rest of it may be easily found, by drawing the curve between them with a slip of thin wood, or

any other such means of producing an easy curve.

116. Charts, on similar principles, are used for many purposes; for example, there are biographical charts, showing the periods when eminent men appeared, and the relative length of their lives. They are also used for representing revenue of any kind, which generally forms an undulating line, as does also the charts of the heights of the barometer, or the temperature indicated by the thermometer. The heights of mountains, the tides—in short, they may be considered as merely scales of equal parts, and, of course, are applicable to all subjects capable of being represented by numbers.

117. In order to give a more distinct comparative view of the tables of pitches, in the “Essay on the Teeth of Wheels,” I shall lay their contents down in one chart, but previously, I shall here collect all these tables, and give some further explanation of their mode of construction,

Tables of Pitches of Wheel Work.

(SEE APPENDIX TO ESSAY ON TEETH OF WHEELS, Art. 90—95.)

TABLE I.*

Velocity of the Pitch Line being 3 Feet per Second,
and breadth of Teeth 9 Inches.

W	X	Y	Z
Pitch in inches.	Breadth of teeth in inches.	Value of strength in horses' power.	Value of strength in horses' power.
4	9	16	12
$3\frac{1}{2}$	9	12.25	10.5
3	9	9	9
$2\frac{1}{2}$	9	6.25	7.5
2	9	4	6
$1\frac{1}{2}$	9	2.25	4.5
1	9	1	3

TABLE II.

The Velocity being 3 Feet per Second, and the breadth of the Teeth double each Pitch.

W	X	Y	Z
Pitch in inches.	Twice the pitch in breadth.	Value of strength in horses' power.	Value of strength in horses' power.
4	8	14.22	10.66
$3\frac{1}{2}$	7	9.53	8.17
3	6	6	6
$2\frac{1}{2}$	5	3.47	4.16
2	4	1.77	2.65
$1\frac{1}{2}$	3	.75	1.5
1	2	.22	.66

* Tables I. and II. are for Teeth attached to water-wheels, where liable to be worn by sand and water.

TABLE III.*

The Velocity being 11 Feet per Second, and the breadth of Teeth 8 Inches.

	W	X	Y	Z
Pitch in inches.	Breadth of teeth in inches.	Value of strength in horses' power.	Value of strength in horses' power.	
4	8	81.77	61.33	
3½	8	62.61	53.66	
3	8	46.	46.	
2½	8	31.94	38.33	
2	8	20.44	30.66	
1½	8	11.5	23.	
1	8	5.11	15.33	

TABLE IV.

The Velocity being 11 Feet per Second, and the breadth double each Pitch.

	W	X	Y	Z
Pitch in inches.	Twice the pitch in breadth.	Value of strength in horses' power.	Value of strength in horses' power.	
4	8	81.77	61.33	
3½	7	54.78	46.95	
3	6	34.5	34.5	
2½	5	19.95	23.94	
2	4	10.22	15.33	
1½	3	4.31	8.62	
1	2	1.28	3.84	

* Tables III. IV. V. and VI. are for Teeth properly greased, and free from sand.

TABLE V.

The Velocity being 3 Feet per Second, and breadth of Teeth 8 Inches.

W	X	Y	Z
Pitch in inches.	Breadth of teeth in inches.	Value of strength in horses' power.	Value of strength in horses' power.
4	8	22.30	16.72
3½	8	17.07	14.63
3	8	12.54	12.54
2½	8	8.71	10.45
2	8	5.57	8.36
1½	8	3.13	6.26
1	8	1.39	4.17

TABLE VI.

The Velocity being 3 Feet per Second, and breadth double each Pitch.

W	X	Y	Z
Pitch in inches.	Twice the pitch in breadth.	Value of strength in horses' power.	Value of strength in horses' power.
4	8	22.30	16.72
3½	7	14.93	12.79
3	6	9.4	9.4
2½	5	5.44	6.53
2	4	2.78	4.17
1½	3	1.17	2.34
1	2	0.34	1.02

*Reference to Table I, Art. 90. in the Essay
on the Teeth of Wheels.*

118. Column X is omitted in Tables I. III. and V. being only a repetition of the same breadth for all the pitches of each table, but as being perhaps plainer, they are inserted here.

The numbers in column Y are found by squaring the pitch in column W.—(See Proposition I. p. 150; see also p. 175.)

EXAMPLE.

The square of 4, (the pitch in inches) = 16, the value of strength in horses' power.—(See the first line of table.)

The column Z is found by inverse proportion.—(See Prop. II. p. 152.) taking three inches, (the standard pitch,) always as the first term, the pitch column, W, as the second, and the horses' power, in column Y, as the third term.

EXAMPLE.

In. In. Horses' power. Horses' power.

3 : 4 :: 16 : 12 (See first line of table.)

3 : 1 :: 1 : 3 (See last line of table.)

Reference to Table II.

119. The numbers in column Y are found here by direct proportion from Table I. nine inches (the breadth in Table I.) being always the first term of the proportion ; the horses' power in Y, Table I. the second, and the breadth in X, Table II. the third term.

EXAMPLE.

In. Horses' pow. In. Horses' power.

9 : 16 :: 8 : 14.22 (See Table II. line first.)

Column Z is found, as in Table I. by inverse proportion, 3 inches, (the standard pitch,) being always the first term. Thus,

In. In. Horses' power. Horses' power.

3 : 4 :: 14.22 : 10.66 (See first line of table.)

Reference to Table III.

120. The numbers in column Y are found by direct proportion, taking 9, (the square of the standard pitch of three inches,) as the first term, and the square of the pitch in W as the second term.

EXAMPLE.

As 9 (the square of 3 inch pitch)

Is to 16 (the square of 4 inch pitch,)

So is 46 horses' power (the value in column Y of 3 inch pitch)

To 81.77—(See first line of table.)

Column Z is found by inverse proportion, as in former tables.

EXAMPLE.

In. In. Horses' power. Horses' power.

3 : 4 :: 81.77 : 61.33.—(See 1st line of table.)

Reference to Table IV.

121.—The numbers in column Y are

found here in a manner similar to Table II.
by direct proportion.

EXAMPLE.

In. In. Horses' power. Horses' power.
8 : 7 :: 62·61 : 54·78.—(See 2d line
of table.)

The numbers in column Z are found, as
in the former tables, by inverse proportion,

In. In. Horses' power. Horses' power.
3 : 4 :: 81·77 : 61·33.—(See 1st line of
table.)

Reference to Table V.

122.—The numbers in column Y of this
table are found by direct proportion from
column Y of Table IV. 11 feet (velocity per
second,) being always the first term, and 3
feet (velocity,) the second term.

Ft. Ft. Horses' power. Horses' power.
Thus, 11 : 3 :: 81·77 : 22·30.—(See 1st
line of table.)

Column Z is found by inverse proportion,
as in all the former tables :

Thus, $\begin{matrix} \text{In.} & \text{In.} & \text{Horses' power.} & \text{Horses' power.} \end{matrix}$
 $3 : 4 :: 22.30 : 16.72.$ —(See 1st line of tables.)

Reference to Table VI.

123.—The numbers in column Y and Z in this table, are found from Table V. in the same manner as those columns in Table II. are formed from Table I. The only difference in Table VI. from Table V. is that which arises from the difference of the breadth of the teeth.

Explanation of the Chart.

124. The scale on the line A B represents the pitch in inches.

The scale on the line A C the horses' power; a single example will probably be sufficient to illustrate the use of the chart.

Suppose the pitch to be $3\frac{1}{2}$ inches, let it be required to find the horses' power to which that pitch is equal when moving at 3 feet per second, in situations where pro-

perly greased and free from sand—observe, where the line from the pitch $3\frac{1}{2}$ intersects the curve Z of Table VI. perpendicular to the point of intersection, on the line A C, will be found 12·79 on the scale or the horses' power to which $3\frac{1}{2}$ pitch, when the teeth are 7 inches broad, is equal, after making allowance for the length of the teeth. (See Table VI. also Essay on Teeth of Wheels, p. 184.)

Observations.

125. 1st, It will be observed, that the curves Y and Z intersect each other, for all the tables on the pitch line marked 3 inches, because that is the standard. (See 1st Essay, p. 172.)

126. 2nd, The curves, continued from 1 inch pitch downward, unite in the points marked O, being the commencement of the scale of pitches, and this part of the curve gives the horses' power equal to any fraction of an inch.

127. 3d, It was already observed, p. 175, that durability as well as strength, should be considered in this investigation. The true proportion, therefore, may be somewhere between the curves Y and Z, but nearer to Z than Y. For although long teeth will be more easily broken than short ones, yet while they do not break, the strain being generally diffused over a greater number of teeth, they will wear longer.*

128. The pitch is laid down on A B, real measure, so that if the pitch should happen to be fractional, it may be taken by a pair of compasses and applied to the chart, which will at once indicate the power to which it may be equal at certain velocities.

* See Art. 51. (Ed.)

ON

THE SHAFTS OF MILLS.

ESSAY II.

P R E F A C E.

It is my wish to make these Essays useful to operative Mechanics, and to save Engineers and Managers of Manufactories the trouble of much explanation in giving directions to foremen and others, who are to carry their ideas into effect. They will, I hope, give such workmen some notion of the principles on which their work should be conducted; and, however much some of them may be disposed to ridicule theory, yet it has always been found that Machinery erected upon true principles, is ultimately the most economical.

Although I have endeavoured to adapt the style of these Essays to the comprehension of merely operative Mechanics, I trust that men of science will not deem them entirely below their notice, those at least whose situation has precluded them from actual practice.

Persons of this description I have known in great difficulty, from want of information about contrivances which are familiar to every Millwright. Nor is it difficult to account for this, for there is hardly any thing to be found in books with regard to many of the subjects of these Essays. As the path then is almost new and untrodden, I hope to meet with indulgence from the public.

The following extract from a respectable periodical publication, appeared to me so applicable to the subjects of these papers, that I have taken the liberty of transcribing it.*

“A country in which Manufactures are extensively established, and conducted with spirit, as in Britain, becomes by degrees a country of Machinery. For inventions to diminish the quan-

* Eclectic Review, Dec. 1806. Art. XII.

tity of human labour employed, will be more ingenious in construction, more powerful in operation, and of more general use, in proportion to the necessity of furnishing a greater quantity of commodities at moderate and equable prices. The bodily exertions of workmen in whatever branch of labour have their limits, and excessive efforts, if unduly prolonged, irremediably destroy the health and vigour of those who pursue them. But machines may be continued in activity day and night, week after week, and month after month; having in themselves no life which suffers a sensible consumption, no principle of activity whose energy requires a pause to effect its recovery or renovation.

“ We have seen the Manufactures of our own country solicit the aid of every hand that could be spared from its Agriculture, and seek in distant lands for labourers of every age to supply the mill or to throw the shuttle. We have seen ingenuity exerted to its utmost, to contrive and to construct those machines which these labourers were to superintend and assist. We remember the time when these constructions were the dread and the hatred of the Manufacturers, but we believe the most ignorant workman of the present day acknowledges

their utility, and would with difficulty be induced to relinquish that very implement which his father or grandfather would have gladly committed to the flames.

“Considering then the importance of Machines to shorten labour, and the number of persons who are interested in them, as proprietors, as inventors, or as constructors, it is wonderful that so little has hitherto been communicated on this subject by the medium of the Press.

“The process towards perfection in complicated Machinery is perhaps too generally the reverse of what might be expected. When practice has shewn the importance of a machine, science takes it up, investigates its principles, analyses its movements, and connects them by the assistance of mathematical precision.

“Mathematicians are seldom inventors, and workmen are rarely men of science, yet the mutual assistance of study and practice is necessary, to perfect the subject which each is intent on improving.” It is my aim, then, to come between these two classes, and to make

them better acquainted, and more useful to each other. How far I have succeeded in the attempt must be left to the determination of time.

In common with all writers on similar subjects, I have experienced considerable difficulty to find precise technical words to express the different parts of Millwork. Those which are used by Millwrights in different districts being very different from each other. I hope, however, that the explanations which I have given of the terms, will make them sufficiently clear.

With regard to this particular Essay on the Shafts of Mills, I have treated the subject in a manner similar to that which I followed in the first Essay in the Inquiry into the Strength and Durability of the Teeth of Wheels. For the reasons then given, p. 142, I have not entered into the demonstrations of the elementary propositions which serve to guide in the Inquiry. I have been at pains to collect and arrange facts respecting Gudgeons and Journals in actual use, upon which to ground calculations. This method I consider as being much more certain than grounding calculations on insulated experiments made on the cohesive strength

of materials. These, however, I confess to be of great value, and in some cases I have endeavoured to apply them. I hope that the various Tables which I have given in the course of the Essay, will be of great use to the Millwright in finding without trouble the sizes of Gudgeons and Journals for any case that may occur in practice, and that the principles on which they are formed are laid down in so plain a manner that he will easily understand and apply them. These Tables may be considered as certain great lines drawn to guide the Millwright in his work, and even allowing they may not be absolutely true, he may find from experience how near they may be approached with safety.

I take this opportunity of acknowledging the advantage which on this, as on former occasions, I have derived from the assistance of my friends.

Glasgow,

DEC. 10, 1808.

ON

THE SHAFTS OF MILLS.

INTRODUCTION.

To proportion the diameters of axles to the stress they have to bear, is in mill-work of great practical importance. On the one hand, if the shafts be made too weak, it is evident they must soon give way; and on the other hand, if made too strong, they occasion not only unnecessary expense in the construction of the machinery, but what is in most cases still worse, a waste of power from unnecessary friction. It is therefore desirable, that the millwright should have some rules to guide him in this very important part of his business; a part which, I believe, has

hitherto in most cases been conducted entirely at random.

I shall endeavour, therefore, in this Essay, to give such a practical view of the subject as shall enable the millwright to proceed with greater certainty.

Until of late years most of the shafts used in millwork were constructed of timber. The use of cast-iron in this and other parts of millwork, however, has now become almost universal. For this improvement we are perhaps indebted to those who are engaged in the cotton manufacture. After Arkwright's invention, it became a great object with them to save time in the erection of machinery, and to render it as durable as possible; for every stoppage was attended with great loss, by throwing idle the numbers of people necessary in cotton mills.

Besides the expense attending the repair, what had perhaps still more weight

with them was, that the profits at that period on cotton spinning, were almost unparalleled in any other branch of manufacture.

Another circumstance which tended very much to the advancement of millwork, arose from Mr. Watt's improvement of the steam engine. This enabled cotton-spinners and other manufacturers who required power to work their machinery to carry on their business in towns. Hence power and people might, without trouble, be concentrated on the most eligible spot, and the great expense and disadvantages avoided which are attendant on colonizing the remote situations in which powerful falls of water are commonly found. I shall not here enter into the question of health and morals ; but certainly the unprecarious supply and steady exertion of such numerous hands, not only rendered the people more expert in all their operations, but tended very much to the improvement of their machinery.

The introduction of cast iron then may be considered as a kind of new era in the history of mills. Without the use of this material, it would not have been possible with the same number of operative mechanics to have constructed one tenth of the machinery which has of late years been erected in Great Britain.

CHAP. I.

SECTION I.

General Description of Shafts.

129. The axles used in millwork are commonly denominated, when of a large size, *shafts*; those which are smaller, are usually called *spindles*. Thus, for example, we say the shaft of a water-wheel; the spindle which carries the millstone of a corn-mill.

130. When shafts lie in a horizontal direction, they are called *lying* or *horizontal* shafts; when vertical, they are termed *upright*, or *vertical* shafts.

131. Shafts are usually made of wood, or of iron. Large wooden shafts are generally made either of solid oak, or are built of fir-logs. The scarcity of large oak occasioned the built shafts of fir to come

into more general use. The latest improvement made on wooden shafts, was that of having what are called cross-tailed *gudgeons*.* Before that improvement, it was attended with very great trouble and expense, to keep the gudgeons from becoming loose in the shafts. Indeed, it was found impracticable to keep them fast for any considerable time.

132. Fig. 1 represents a wooden shaft, with the gudgeons in use previously to the last improvement. They are called *laid in gudgeons*. ABC is the gudgeon somewhat in the form of the letter T. One of the tails, C, was let into a mortice, and the rest of the gudgeon sunk into its place in the centre of the shaft. In order to accomplish this, it was necessary to cut out the part, from B to D, Plate II. Fig. 1, No. 2. After the gudgeon was laid in its place, the vacant part was filled up by the piece

* The *gudgeon* is the arbour or spindle on which the shaft turns.

of wood D E, Fig. 1, No. 1. The shaft was then hooped, and the end of it driven full of wedges, in order to fasten the gudgeon. This gudgeon is shewn in perspective, Fig. 1, No. 3.

133. Fig. 2 represents a wooden shaft, with cross-tailed gudgeons. This kind of gudgeon is made of cast-iron, and being thin in the cross tails, let in from the end of the shaft, it leaves the wood much more entire than the *laid-in gudgeon*, while its cross-arms take a much firmer hold. After it is let in, the hoops are driven on the end of the shaft, when warm, and lay firm hold of the ends of the cross-tails. The wood is then wedged up, which makes the gudgeons perfectly fast. Fig. 3 is a perspective view of a cross-tailed gudgeon, and Fig. 4 its profile. It is cast with the round part undermost; for which reason the pattern must have a taper, to make it rise out of the sand. This taper has, in the cross-tails, another very important use, that of giving the gudgeon the advantage

of *dove-tailed joints* with the timber of the shaft when it is wedged up. Instead of wrought-iron hoops, cross-tailed gudgeons sometimes have a hoop of cast-iron cast along with the tails, as represented by Fig. 5.

[A 133.] When it is considered that the direction of the stress which tends to loosen the gudgeon in a wooden shaft is continually changing, and that such action is exerted upon wood, a material that is so very easily permanently compressed; it will not be wonderful that it should have been found difficult to render them firm and lasting. The last method, viz. that where the hoop is cast along with the cross tails, seems to be far preferable to the other; but perhaps it seldom happens that the hoop part is of sufficient length. It may be proved, that when the diameter of the shaft is sufficient for the strain upon it, the length of the hoop should be equal to the square of the diameter of the shaft divided by the length of the shaft; other-

wise there will be no certainty of the gudgeon remaining permanently fixed.

[B. 133.] An improved method of fixing gudgeons is described in the Transactions of the Society of Arts, &c. vol. xxxi. p. 223; it consists in casting the gudgeon with cross arms, which fit into proper notches in an octagonal box of cast iron that has been previously fixed upon the end of the shaft. The arms of the gudgeon are retained in their places by screw bolts.

In Plate [IV. A,] Fig. 1, 2, and 3, A A represents the end of the wooden shaft, which is supposed to be made of an octagonal form. B B is the cast-iron box accurately fitted on the end of the shaft, and wedged tight. The end of the box has a projecting flanch *aa*, with four notches to receive the cross arms *bb*, *dd*, of the gudgeon C. These cross arms are firmly fixed to the box by four screw bolts, which pass through the flanch, and the ends of the cross arms. The section, Fig. 3, shews

the box B B on the end of the shaft, with the gudgeon C, and its cross arms separated; to explain a further precaution which is necessary for strength. This precaution consists in the cross arms having projections *e e*, which enter the end of the box, and keep the gudgeon true to its centre, and prevent any lateral strain on the bolts. When the gudgeon of a wheel is fitted according to this method, it can be easily removed when it is so far worn that a new one is necessary; and the new one inserted without injury to the end of the shaft.

This improved method was invented by Mr. Robert Hughes.

It is obvious that the length of the box should be regulated by the rule stated in Art. [A. 133.]* The real advantage gained by this mode of fixing seems to be, that of retaining the end of the wooden shaft more perfect, with the means of

* The rule supposes the shaft to be proportioned to the stress upon it.

renewing the gudgeon, without injury to the shaft. (Ed.)

134. Cast-iron shafts are sometimes made hollow cylinders, and sometimes they are made solid, and of various figures. It is demonstrable, that a hollow cylinder is much stronger, with the same quantity of matter, than it would be if made into a solid of the same length. This law is very observable in the beautiful economy of nature ; for instance, the stalks of plants, the quills of birds, the bones of animals. But, in the works of art, numberless obstacles to perfection continually occur. In this particular case, the expense of making small shafts hollow, would be very great ; and another objection is, the difficulty of making such castings perfect. Shafts of a small diameter are, therefore, commonly made solid.

135. Fig. 6, Plate II. represents a cast-iron cylindrical shaft. It consists of three parts, the body, A B C D, and the two gud-

geons, A E C, and B P D G. The gudgeons are turned, and carefully fitted into the ends of the body, which is bored and turned to receive them. They are then fixed with screw-bolts, which pass through the flanches, as may be seen by the figure.*

This kind of shaft will obviously be variously constructed, according to circumstances. That represented in the figure was made for a cast-iron water-wheel. The use of the small projections H, H, &c. is to prevent the eye of the arms from shifting round on the shaft. When cylindrical shafts are not used, what are called feathered shafts are often adopted.

136. Fig. 7, Plate III. represents this construction of a shaft. It probably took its

* In this construction, the resistance to twisting depends entirely on the bolts, and some difficulties appear to be encountered in casting without corresponding advantages, either in strength or beauty. (Ed.)

name from its resemblance to the feathered part of an arrow. It may be here remarked, that shafts of this species, as often constructed, are by no means calculated to withstand the twist brought upon them by the strain of the machinery. From the breadth of the feathers, their strength to withstand lateral pressure, is, no doubt, considerable; but wanting substance between the feathers, they are liable to continual tremour. Where feathers are applied to shafts, I would prefer keeping the body of the shaft fully as strong as the gudgeon, or *journal**, and apply the feathers merely to prevent bending in the middle, as Fig. 7, No. 4. But the simple square, Fig. 8, is more easily made, and, I believe, has been found in practice, at least as advantageous as any other form that has been tried for solid shafts.†

* *Journals*, or *journeys*, are gudgeons subject to torsion.

† It is easily proved, that the best form for a revolving shaft is a cylinder, and that in any other form the

Having given this general account of shafts, I come next to consider the causes from which the stress on them arises.

SECTION II.

Of the Kinds of Stress to which Shafts are subject.

137. There are two kinds of stress to which shafts are liable: first, *lateral stress*, by which they may be broken across: secondly, stress arising from *torsion*, by which they may be wrenched or twisted.

flexure will be irregular, and consequently produce irregular wear on the gudgeons and brasses; but when a shaft is to be adapted for placing wheels on any part of its length, a square section is convenient; in all other cases, the section ought to be circular with projections, as at H, H, Fig. 6, Plate II. By making four of these projections continue throughout the length upon a cylindrical shaft, all the advantage and convenience of a square one would be obtained, with very little irregular flexure. The projections should not be greater than is necessary to fix the wheels firmly on the shaft. (Ed.)

All horizontal shafts are liable to the first kind of stress, viz. lateral stress; and some have no other strain whatever; as for instance, a water-wheel shaft, where the motion is communicated from teeth, on the shrouding. See Plate III. Fig. 9.

In Fig. 10, the stress on the upright shaft, arises from torsion only; excepting what may proceed from the inaccuracy of the teeth of the wheels, which, if great, will occasion a considerable lateral thrust.

A vertical shaft, which gives or receives motion, by means of a pulley, has thereby a lateral pressure brought upon it.

In Fig. 11 and 12 the stress is compounded of *lateral pressure*, arising from the weight of the wheels A and B, and that of the shaft itself, and of the torsion or twist produced between the wheels A and B.

The following letter, with which Mr. JOHN ROBERTON, engineer, favoured me, as it relates to the subject of this Essay, and contains much useful matter, will, I hope, be acceptable to the reader:—

DEAR SIR,—In answer to yours, of the 7th, I shall throw out a few hints, which occur to me respecting the stress on gudgeons, shafts, &c. It is demonstrable, that by a judicious arrangement of wheels and pinions, in many cases much of the stress and friction may be avoided. This is a doctrine in practical mechanics of very great importance, when we consider, that in many cases almost the whole of the impelling power is expended in overcoming the friction of the machinery.

138. Let A, Fig. 1, Plate IV. be an over-shot water-wheel. Let the line ab , represent the line of direction of the centre of gravity of the water in the buckets. On the extremity of this wheel let there be a toothed wheel acting into the pinion B. It is obvious, that independently of the weight of the wheel, that the whole weight of the water will be supported by the axis c , and the teeth of the wheel at d ; and the weight which each will sustain, will be in the ratio of ce to ed . That is, the

weight on the gudgeon will be as the distance ed , while that on the teeth will be as ce ; or if the radius of the toothed wheel were ce , the teeth would sustain the whole weight of the water, leaving no weight on the gudgeon but that of the wheel. Again, let the wheel B , be removed to C , it is evident that the gudgeon c , would have to sustain the weight of the water, and a great deal more. That is, the weight on the gudgeon would be increased as e to ec ;* so that it is evident, the stress and friction of the gudgeon must depend, in a great measure, on the size of the toothed wheel attached to the water wheel, and to the situation of the pinion B .

139. Again, let there be a wheel at A , Fig. 2, fixed on the end or middle of a shaft working into the wheel or pinion B , of any size.—Let the teeth move in the direction ab . It is evident, that the

* The true relation of the stress in the two cases, is
 $ed : \frac{2(dc)^2}{dc+ed} \cdot$ (Ed.)

gudgeon or shaft will tend to move in the contrary direction, that is, in the direction *c d*, and with the very same force that the teeth act upon each other, as action and reaction are equal and in contrary directions, and for the same reason, the gudgeon of the wheel B, will tend to move in the direction *e b*, with the very same force, that is, the same force as the action of the teeth on each other. Indeed, in any single pair of wheels, of whatever form or construction, the tendency to break or bend the shaft, or cause friction, is the same as the action of the teeth on each other.

Now, if the above wheels were made of a double size, it is evident that the acting power on them would be only one half, and consequently, one half of the strain to break the shaft or cause friction.*

* The object of this remark is, apparently, to show the superiority of large wheels ; but it is clear that the acting power would be the same with the same first mover ; and if the resistance be diminished on one shaft, another must be added to give the proposed velocity to the working point. (Ed.)

140. In the case of an intervening wheel, the force or tendency to break the shaft depends on the situation of such intervening wheel. Thus, if it be placed in a direct line betwixt the centres of the conducted and conducting wheels, as at A, Fig. 3, the shaft or gudgeon will tend to move in the line ab or ba , (according to the direction of the conductor,) with double the force of the action of the teeth.

If the axis of the intervening wheel form a right angle with the axis of the other two wheels, the force to break the shaft will be to that of a pair of single wheels, or the action of the teeth on them, as the diagonal of a square is to one of its sides. That is, the direction of the force, and its intensity, will be represented by the diagonal, it being evident, from the well-known laws of mechanics, that by the action of the wheel B, Fig. 4, No. 1, the centre of the intervening wheel A, would tend to move in the line ac , and by its action in the wheel e , it would tend to move in the line ad . Let ad and ac represent the forces

in these directions, complete the square or parallelogram, the diagonal of which will both represent the force and its direction. See also Fig. 4, No. 2, and No. 3.

141. On the other hand, if a wheel be placed betwixt two others, as A, Fig. 3, where A is the conductor, the teeth of which acts with equal force on each of the wheels B and C, it is evident, that the strain is wholly taken off the shaft, the forces being equal and opposite each other*; and in Fig. 5, where A is supposed the conductor, and B and C the conducted wheels on which the teeth on each bear equally,—it is plain from what has been already stated, that the direction of the forces will be da and ac , and letting ad and ac represent

* The shaft ought not, however, in any case, to be entirely freed from pressure in this manner; because its motion will not be so steady and regular as when there is some considerable pressure on the gudgeons. And since there is nothing more injurious in machinery than a hobbling, unsteady motion, this point requires the Engineer's most careful attention. (Ed.)

the direction and intensity of these forces, and completing the parallelogram, we have the diagonal *ab* to represent the compound direction and intensity of the force.

142. There is another point that is worthy of attention: that is, the place on the shaft where the wheels are fixed. If a wheel is put on at the end of a shaft to drive any other or others, it is clear, that the whole or nearly the whole of the stress will be at the end of the shaft or journal. If the wheel be placed in the middle of the shaft, the strain to break it will be greatest at that part, but the force will be resisted equally by each journal. Indeed, on whatever part of the shaft a wheel is placed, at that very place is the greatest (cross) strain on the shaft, and the force on each journal will be in the inverse ratio of the distance of the wheel from the ends of the shaft.

In Fig. 6, let *A* represent a shaft, either upright or lying. If a single wheel, as *B*, fixed upon it, drive two pinions, as *CD*, directly opposite to each other, the shaft

or journal will not be affected thereby,* but if two wheels are placed on the shaft, driving each a pinion as in Fig. 7, both journals will be affected, and the greatest strain to break the shaft will be at the arms of the wheel, or close to them, and betwixt the arms and journal. In this case, the middle point of the shaft, as at A, being in a state of contrary pressure, and therefore no strain on that part, it may be considered as a lever on each side of A; this being the fixed point, and the greatest strain on the shaft being at the wheels. The strain on the journal will be inversely, as the distance of the wheels from the middle point A, it being understood that the wheels, pinions and resistances are all the same.

143. Again, if two wheels are placed on a shaft, and two pinions, both on the same side of the shaft, the journals must support the strain or force of the action of the teeth

* See note to Art. 141.

of both wheels, or indeed whatever number of wheels is on a shaft; and working into pinions on one side, the journal of that shaft must support a pressure equal to the whole of their action into the teeth of the wheels or pinions into which they are connected; and the strain on the shaft to break it, depends on the situation of the wheels, as they may be placed on the shaft: it being understood, as formerly, that directly at the place where the wheels or pinion is fixed on the shaft, that it must support a pressure equal to the pressure of the teeth of the wheel in its corresponding wheel or pinion. And according as these pressures combine, or act in a contrary direction to each other, so must the strain on the journals, or tendency to break or bend the shaft be.

Hence the importance of placing wheels and pinions, so that their actions on each other may be in contrary directions, or so as to avoid, as much as possible, the strain on the shaft or journals.

144. In lying shafts, where circumstances may require, it may be advisable to have the main or heaviest shaft on the lift of the wheels, as this will take off a considerable deal of their weight or friction on the journals. The other shaft will naturally be on the fall of the wheel, (which is supposed in this case much lighter than on the main shaft,) and will be prevented from jolting upwards, from both its own weight, and a pressure equal to that on the teeth of the wheels, being supported by the journals. It being evident that whatever the wheels are, that the same pressure that is on the teeth of the wheels will be equally the same on both shafts, the one tending to increase the weight of the shaft on the journals, and the other to diminish it.

145. In the course of my business, I have made several observations on the foregoing subjects. Twenty years ago, a case of this nature occurred, of considerable im-

portance, at the flour mill erected near the Slitt Mills of Patrick. The water wheel, 16 feet diameter, makes about 10 or 11 revolutions per minute, driving, in general, two pair of stones ; the pit wheel about $6\frac{1}{2}$ feet diameter. The consequence was, that the machinery, framing, &c. were not of sufficient strength to bear the force applied by the pit wheels, (though they were very strong, and well executed for ordinary cases,) the shaft, framing, &c. was in a high state of tremour, the machinery working in a rough and straining manner, and the wooden teeth (5 inches broad, pitch about $4\frac{1}{2}$) could not stand for any length of time. The mill was altered, by enlarging the pinion, to let the wheel run at 14 or 15 turns per minute ; and I believe the pit wheel enlarged about one foot diameter ; afterwards the mill wrought very well.

In common corn-mills, and many others, there is a great deal of the impelling power lost by the smallness of the pinions, &c. which causes a great friction on the shaft

or spindle, as well as by the friction of the teeth. Were both wheel and pinion encreased in diameter, much advantage would arise, not only in saving of power, but in the tear and wear of machinery. It is on this account that a corn mill, constructed in the double way, other circumstances being the same, performs much more work than in the single way.

146. In short, more things of this nature take place in machinery, than the most of operative mechanics, or even philosophers, are aware of; and I firmly believe, that, with a knowledge of the affecting causes, a vast deal might be done in saving power. For example, in a cotton mill, the main shaft generally makes from 40 to 50 revolutions per minute. Let the weight of the shafts and machinery, the size of the journals, and the effect of the wheels on these shafts, and friction of journals, be taken into the account, on the one case, and let the main shaft be supposed to be reduced to half of the former velocity, that is, from

20 to 25 turns per minute, and let the strength of the shaft, together with a due proportion of wheels, be augmented, so as to preserve the same firmness in every part of the machinery as in the former, and that the ultimate part of the machinery may be brought to the same speed as formerly, it will be found on investigation, that in the last case there will be a considerable saving in the first, or impelling power.

It must not be lost sight of, that, by reducing the velocity of the main shaft, that a judicious increase of the diameters of the wheels thereon is absolutely necessary. Indeed, without strict attention to matters of that kind, the effect of any alteration that may be proposed or made is very precarious. A due regard to the proper diameter of wheels, according to the work they have to perform, is a matter of very great importance; and it will be found, on general inquiry, that in most cases of machinery, it would be prudent to have the wheels and pinions of large

diameter; and, as I said before, by increasing their size, the force, strain, and friction on the shafts and journals are diminished in the same ratio.

Suppose, in a mill, that a range of lying shafts, of 80 or 100 feet long together, with wheels, &c. fixed on them, weighed 7000 lbs; the journals 4 inches diameter, making 46 turns per minute; the surface of the journal would at that rate move at about 50 feet per minute: and supposing that the friction was equal to one third of the weight, we should have $7000 \div 3 = 2333 \times 50 = 116650 \div 44000 = 2.66$; that is, nearly $2\frac{3}{4}$ horses' power expended in overcoming the friction of these shafts.*

* That is, valuing the horses' power at 44000 lbs. 1 foot per minute. See Essay on Teeth of Wheels, Art. 79.

The quantity of friction is much over-rated, but the loss of power would be very considerable on the lowest estimate; and therefore, the proper situation for the first mover in a system of machinery is of some importance. (Ed.)

Again, were these shafts and journals extended to 5 inches diameter, and their number of turns reduced to one half of the former; that is, to 23 turns per minute, the surface of the journal would move at the rate of 31 feet per minute, and the weight of the wheel and shafts encreased one half, or say, to 10,000lbs. we should have $10,000 \div 3 = 33.33 \times 31 = 103323 \div 44.000 = 2.348$, that is, nearly $2\frac{1}{2}$ horses' power; so that in this last case there would be a saving of something more than three tenths of a horse's power, and the machinery would be, in every respect, improved.

147. In a horse-gin, where a pinion is driven by a toothed wheel on the gin, the friction, or strain on the journals, depends on the situation of the horse-beam; at any moment of time when the lever or horse-beam is above or below the pinion, the friction on the shaft will be the least, and when in the opposite direction, the friction and force on the shaft will be the greatest,

and the general friction will be as the size of the main wheel; that is, the smaller the main wheel is, the friction and force on the shaft will be the greater, and the larger this wheel is, the friction, &c. will be the less; it being understood that I mean the friction or strain on the gudgeon, or axle, on account of the re-action of the teeth of the wheel, and not the strain on the shaft, to twist it, nor any other friction or strain whatever.

148. In a water-wheel turning machinery, in many cases, it is most advisable to have the toothed wheel of the same diameter. In this case, whatever power or force is applied to the wheel, the same must be resisted by the teeth of the wheel; and it also follows, of course, that whatever is the size of the wheel, or pinion which is driven by the main wheel, that the very same strain is on the shaft; that is, the same as on the teeth; and the shaft must be sufficiently strong to withstand the pressure. But another circumstance

occurs, that by increasing the size of the pinion, there must necessarily be increase of the diameter of the shaft to withstand the twist. The shaft by no means, however, keeps pace with the augmentation of the wheels, but is only as the cube-root; for instance, a water-wheel of 16 feet diameter would work very ill into a pinion of 12 inches, suppose its shaft or journal to be 3 inches diameter. Again, let the pinion be increased to 2 feet diameter, a shaft of $3\frac{3}{4}$ inches will be as sufficient to withstand the twist, the friction on the journals will be much less, and the strength to resist the strain will also be much increased. I do not see how general rules can be laid down with accuracy in these things. It is the particular circumstances of the case that will guide a mechanic in the construction of any piece of machinery; and if he be not well acquainted with these, it cannot be expected that his schemes will be well arranged.

The above observations are no doubt at variance with the opinions of those who

are continually holding up the simplification of machinery. Instance Mr. Fenwick's "Essay on the Simplification of Machinery."* In many cases it is prudent to make machinery of a more complex nature than it is sometimes constructed; and in order that it may be easier driven, that the tear and wear may be lessened, as well as the ultimate expense and trouble of attending it.

The foregoing are only a few general hints. To go minutely into the subject would require more time and thought than I could at present bestow; but if any thing is in them from which you may draw any

* The maxims of Mr. Fenwick consider friction only, and are so far correct; but the wear and tear, and friction of teeth he has not taken into account. His 6th maxim, that "Small wheels are equally as generative as large wheels, if the same ratio of size be preserved," is true only within certain limits, (see Art. 28.) but these limits being assigned, his other maxims hold till the stress on the moving parts is less from a greater number of small wheels, than from fewer large ones. (Ed).

useful inference, it will be very agreeable to me.

I am, Sir,

Your's respectfully,

JOHN ROBERTON."

Tradestown, Glasgow,
SEPT. 12, 1809.

149. In the essay to which Mr. Robertson in the foregoing letter alludes, Mr. Fenwick infers, (p. 64.) that "*the most perfect machine is that which operates with the fewest moving parts.*" But Mr. Fenwick seems to have been misled here by a desire to generalize. Simplicity is, no doubt, a most desirable quality in a machine, provided it can be obtained without making a sacrifice of power or of durability. A sledge has fewer moving parts, and in that sense is more simple than a wheel-carriage, yet no one with truth could say that a sledge is more perfect.*

* The Essay by Mr. Fenwick here alluded to, is wholly confined to the application of wheel-work;

Nor, perhaps, does the *simplicity* of a machine, consist strictly in having *few moving parts*. If the parts of a machine be few, they are perhaps more easily taken in by the eye at one view, which may make them more easily comprehended by the mind, and in that sense be more simple. But in machinery, the kind of simplicity at which we ought to aim, has more regard to the manner *of action* than to the number of the moving parts. Thus, for example, when a weight is to be raised, if one wheel worked by a screw be employed, the machine *consists of fewer parts* than two wheels and two pinions, applied to the same purpose. But in this last case, the *manner of action* is really more simple; for the action and resistance are directly opposed in the same line; whereas, in the case of the screw, the action is oblique, and experience shews

consequently the remarks in this and the following paragraph are not applicable to his Essay. (Ed.)

that it has much more friction, and much less durability.

What is here said of *manner of action* is applicable in comparing machines, consisting each of trains of wheels and pinions; for the most durable, by longest maintaining the true figure of the teeth, will ultimately be the most simple in the manner of its action. It is evident, that when the teeth become much worn, that the *manner of action* becomes proportionably more oblique and less simple.

Respecting the durability of the wheel-work of mills, I beg leave to refer the reader to my Essay on the Teeth of Wheels, Ess. I. Art. 73.

[A. 149.] It can be of very little use to give rules for the diameters of gudgeons, or for the strength of shafts, unless they be accompanied with some method of estimating the straining force. Our author has not touched upon this part of his subject; and therefore I have inserted Mr. Robertson's letter in this place, in view

that the information it conveys, respecting the stress on shafts and gudgeons, may be studied in its proper order; and some additional inquiries I shall add in these articles.

Let A B, Plate [IV. A.] Fig. 4, represent a shaft, with one wheel at D, and another at C; these wheels being of any size whatever. If a power act upon the wheel D at the point P, and the resistance be at W, the stress arising from these forces will cause a pressure on both the gudgeons; and the line W P being drawn, cutting the axis at some point E; the stress upon the shaft and gudgeons will be the same as if a force equal to the power and resistance together, were applied at the point E. Hence it is clear that when the wheels differ considerably in size, the gudgeon next the smaller wheel will have to sustain the greater part of the stress.

[B. 149.] But if the resistance were at w , the power and resistance in this case being at the same side of the shaft, the

pressure will be downward on one gudgeon and upward upon the other. For the descending power P is resisted by the support of the gudgeon at A , and by the resistance at w ; but the power not falling between these points, it will tend to raise the gudgeon B .

[C. 149.] Again, if the resistance be at the point C on the upper or under side of the wheel C , the pressure at the gudgeon B will be wholly in a lateral direction; consequently when the impelling power moves with the wheel, the stress on the gudgeons will vary considerably both in intensity and direction.

If the plan of the shaft and wheels be drawn to a scale, it will be easy to compute the pressures in these different cases, and to compare them; and perhaps a young mechanic will feel some pleasure in such comparisons, where he would have been fearful of engaging with a mass of algebra.

Case 1. The power and resistance being at opposite sides of the shaft. Draw

the line $A B$ in the middle of the shaft, and also draw the line $W P$. Then to find the stress upon the gudgeon B , we shall have $A B : A E ::$ power added to the resistance : stress on the gudgeon B ; or $\frac{A E \times (P + W)}{A B} =$ the stress upon the gudgeon B .

Also $A B : B E ::$ power added to the resistance : stress on the gudgeon $A = \frac{B E \times (P + W)}{A B}$.

Case 2. The power and resistance being at the same side of the shaft. Draw the lines $A w$ and $P B$ which cut one another at F . Then, the line $P B$ may be considered a lever with its fulcrum at F ; and to find the stress necessary to keep the gudgeon B down, we have $B F : P F ::$ power at P : stress at $B = \frac{P F \times P}{B F} =$ the stress on the gudgeon B . This stress will obviously be opposed to the weight of the shaft and wheels. Also, $A F : w F ::$ the resistance at w : stress on the gudgeon at $A, = \frac{W \times w F}{A F}$. This stress will be to add to the stress from the weight of the shaft and wheels.

Here it may be remarked, that it is desirable that the greater pressure on any gudgeon, should always, when convenient, be in the direction of gravity, to prevent the unpleasant jolts which take place when the machine is put into motion, when the pressure is upwards upon any of the gudgeons.

[D. 149.] But the preceding cases suppose that the power at P and the weight or resistance at W, are parallel, they are, however, often oblique in respect to one another. Let E, Fig. 5, 6, and 7, on a plan of such wheels, be the axis; let P be that point in the circumference of one of them where the power acts in the direction DP; and let W be the point where the resistance acts in the direction DW. Then DE will be the direction of the stress upon the axis, and if Dc be made proportional to the resistance; or Db proportional to the power; and if the parallelogram Dcab be completed, aD will be proportional to the whole stress upon the axis, which is

obviously greatest in Fig. 5, and least in Fig. 6. Now, make cd perpendicular to DE , then Dd is the pressure on the axis caused by the resistance at the circumference of the lesser wheel; and ad will be the pressure on the axis from the power at the circumference of the large wheel. But it must be remarked, that when the direction of the stress on the axis falls between the direction of the power and that of the resistance, as in Fig. 5, and 7; the whole stress will be either in the direction DE , as in Fig. 5, or ED as in Fig. 7, and proportional to Da . Whereas, if the directions of the power and resistance be both on the same side of the axis, as in Fig. 6, Dd will be the pressure on the axis at the place of the small wheel W , in the direction ED , tending to raise the axis; and da the pressure on the axis at the large wheel P , in the direction DE ; or opposite to the pressure at the wheel W ; while aD is the difference between these pressures, shewing the whole tendency of the axis to rise.

The figures are all drawn to the same

size, and the power and resistance being represented by equal lines in each figure, the advantage or disadvantage of any particular construction will be seen by inspection ; and the reader may easily multiply examples, by drawing more figures.

The relation and intensity of the pressures might have been shewn in a more elegant manner by the arithmetic of sines, but perhaps not so satisfactorily to the minds of most of my readers. And when the rule and compasses are in the hand, the relations are more easily, and with less risk of mistake, ascertained by drawing the wheels and axis as shewn in the figures ; as it merely requires a little knowledge of the composition and resolution of forces.

Thus, make Dc , from a scale of equal parts, equal to the number of cwts. in the resistance at W , and make ac parallel to the power PD , and cd perpendicular to DE . Then Dd , measured from the same scale, is the stress at the centre of the wheel W in cwts. and ad the stress in cwts. at the centre of the wheel P .

The stress from the resistance and power being determined, that which is caused by the weight of the shafts and wheels themselves will be easily calculated. See Art. 181 and the following articles. (Ed.)

The case of lateral stress being the most simple, I shall, in the first place, examine it, confining for the present our attention to the gudgeons only. The bodies of shafts will be afterwards considered.

CHAP. II.

SECTION I.

Of the Strength of Gudgeons, where the Stress is produced by lateral pressure only.

150. The gudgeons having all the weight on the shaft to support, ought to be made sufficiently strong for that purpose; while, to avoid unnecessary friction, they should be made as small in diameter as possible, consistently with sufficient strength and durability.

When we are able to determine the diameters of the gudgeons, or journals, this serves as a foundation for the proportions of the other parts of the shafts.

Although wrought iron will bear a greater weight than cast iron, yet cast iron being not only cheaper, but much more easily formed into convenient shapes, gudgeons are now most commonly made of that ma-

terial. We shall, therefore, in the first place, confine our attention to the diameters of gudgeons made of cast iron. Here it may be proper to state the following proposition :

151. Prop. I.—*Solid cylinders of the same length have their lateral strength as the cube of their diameters**.

That is, if a gudgeon of two inches be sufficient to support a certain weight, a gudgeon of four inches will support eight times as much.

From this law, it is evident, that were all gudgeons made of iron, of the very same quality, knowing the strength sufficient, in any one case it would be easy to calculate what it should be in any other case.

* See Emerson's 4to. edition, prop. 67, cor. 2. Gregory's Mechanics, vol. I. art. 170. cor. 3. There is, probably, a *typographical error* in Dr. Brewster's edition of Ferguson's Lectures, vol. II. p. 24. where it is said, "The diameter of the gudgeon must be proportional to the *square root* of the weight which it supports."

But as there is a great variety, in point of strength, in different kinds of iron; Welch cast-iron, for instance, being stronger, as some think, by one fourth, than that made in Scotland, it is prudent to calculate upon the weakest. Mr. Banks observes,* that, “Iron is much more uniform in its strength, than wood; yet it appears that there is some difference in different kinds of ore, or iron-stone; there is also a difference from the same furnace, perhaps owing to the degree of heat which it has when poured into the mould.”

[A. 151.] The strength of a gudgeon is limited by the strain it will bear without permanent derangement of its structure, for the length is always so small in regard to the diameter that the flexure will be, in all practical cases, insensible.

The calculated stress should include every kind of force acting on the axis or

* Banks's Power of Machines, p. 94.

shaft; and the diameter should be determined, so that the gudgeon would be capable of resisting the whole stress if it were thrown upon the extreme point of its bearing, (see Art. 154.)

When W is the utmost amount of the stress in cwts. and l the length of the gudgeon in inches, from the shoulder to the extreme point of bearing, it is shewn (Essay on Cast Iron, Art. 138.) that $\frac{(112W \times \frac{1}{2}l)^{\frac{1}{2}}}{5} = d$, the diameter of the gudgeon in inches. Or, $0.42 (W l)^{\frac{1}{2}} = d$.

[B. 151.] But an allowance should be made for wear, which will be nearly directly as the stress, and inversely as the length of the gudgeon's bearing; consequently the length of the gudgeon should be greater in the same ratio as the stress is greater. And till some more certain principles of proportioning the gudgeons of a machine so as to be of equal duration shall be found, we may allow one fifth of the diameter as a provision against wear where

no gritty substance is likely to affect it, and one third in all cases where the gudgeons are exposed to gritty matters.

RULE.—In the former case, the rule will become $0.5(Wl)^{\frac{1}{3}} = d$. That is, multiply the stress in cwts. by the length of the gudgeon in inches, and the cube root of the product being multiplied by 0.5, will give the diameter of the gudgeon in inches.

When a gudgeon is likely to wear much from the nature of the machine or its particular situation, multiply the cube root of the product by 0.6, instead of 0.5. Gudgeons of water-wheels may be included in the class which are exposed to considerable wear.

If the stress on one gudgeon be equal to the weight of the wheel, and the wheel be at the middle point, that part of the stress which is produced by the action of the moving power and resistance, must be considered equal to half the weight of the wheel; for only half the weight will bear on one of the gudgeons in this case. But

it often happens that the wheel is considerably nearer to one bearing than the other, and in such cases, the rule of the author would be likely to mislead.

Since our author has given it as a general principle, that the diameter of a gudgeon should be equal to the cube root of the weight supported in cwts. it will be desirable to compare our rule with that principle; first assuming that the weight is actually equal to the stress upon the gudgeon. Now in that case, the rules will be the same when $0.5 l^{\frac{1}{3}} = 1$, or $l = 2^{\frac{1}{3}} = 1.26$.

And in the second rule, when $l = \left(\frac{1}{.6}\right)^{\frac{1}{3}} = 1.185$ inches. Therefore whenever the length of the gudgeon exceeds about $1\frac{1}{4}$ inches, the rule gives the diameter too small. Again, if we take the actual stress to be only half the weight of the wheel in cwts. which is clearly in all cases less than the real stress, the above numbers should be multiplied by the cube root of 2. which will show that the author's rule becomes in

defect again when the length exceeds 1.587 inches. (Ed.)

I shall proceed in this inquiry, on principles similar to those which I have followed in my *Inquiry respecting the Strength and Durability of the Teeth of Wheels*, namely, by taking a number of cases from millwork in actual use, and drawing inferences from them. This method of making inferences, by collecting and arranging facts respecting millwork, I consider as being much more safe and useful, than founding calculations upon experiments, often made on a small scale, and under circumstances very different from those which occur in practice with machinery. I shall begin with considering the gudgeons of water-wheels.

SECTION II.

Of Gudgeons of Water-wheels.

152. In the following table, water-wheels of various weights are collected, and the

diameters of the gudgeons in actual use, stated. The weights of the cast-iron wheels were found, from the weight of the castings, &c. of which they are composed. The wooden wheels are estimated, making allowance for the wood becoming heavier by being soaked in water. I am aware, however, that besides the mere weight of the wheel, other circumstances should be taken into account,* such as the weight of water in the buckets, and the pressure brought on the gudgeon by the resistance of the work, &c. But I shall follow the general result of those cases only, in which the gudgeons have been found sufficiently strong, and, I apprehend, that those extraneous causes on the one hand, will not affect our rules more than the different qualities of cast iron will the strength of gudgeons on the other hand.†

* See Encyclopedia Britannica, article Rotation.

† These remarks render it necessary to say a few words on rules founded on empirical principles, and particularly when the circumstances are not minutely

The water-wheels in the table were all in the middle of their respective shafts. Both gudgeons of each wheel had, therefore equal stress.

153. *Description of the first Table of Gudgeons of Water-wheels.*

Column 1 contains letters to distinguish the wheels.

detailed for those cases on which the rules are founded.

The rule for the strength of gudgeons in the text, supposes the stress to be always proportional to the weight of the wheel, but this supposition scarcely ever corresponds with the truth; consequently, in the cited practical cases, if the stress was not the greatest possible, in regard to the weight of the wheel, and the metal of an inferior quality, this rule may lead us into serious errors. But the practical cases are not described, and therefore we cannot judge from any thing in the text, of the safety of the rule.

When a rule is to be formed by any process, every cause of stress should be considered, and where simplicity is desirable, the stress should be represented by a quantity which is certain to equal it, even in an extreme case. We cannot err greatly, if the error be always on the side of strength. (Ed.)

Column 2 Shews the material of which the wheel is made.

- 3 The diameter in feet.
- 4 The width in feet.
- 5 The kind of wheel.
- 6 The diameter of the gudgeon in inches.
- 7 The weight of some of the wheels, in tons and cwts.
- 8 The weight of some of the wheels, in cwts. and qrs.
- 9 Contains the cube root of the weight.

The use of this column is to compare the several gudgeons with the law contained in Prop. 1. For, were all the gudgeons duly proportioned to the weight they have to sustain, they would be to one another as the cube roots of their weights.

TABLE I. GUDGEONS OF WATER-WHEELS.

1	2	3	4	5	6	7	8	9
	Wheel made of	Diameter of wheel in feet.	Width in feet.	Kind.	Diameter of cast-iron gudgeon in inches.	Weight of wheels in tons and cwt.	Weight of wheels in cwt. and qrs.	Cube Roots of weight in cwt.
A	Wood	24	12	Overshot	7	23	14	7.796974
B	Cast-iron	16	6	ditto	7	12	240	6.214464
C*	Cast-iron	16	8	ditto	6 $\frac{5}{8}$	16	10	6.910423
D	Cast-iron wheel and buckets	32	4 $\frac{1}{2}$	ditto		24	480	7.829735
E	Wood	16	9	ditto	6	10	11	5.953341
F	Wood	12 $\frac{1}{2}$	7	ditto	6	5	14	4.848808
G	Wood	32	11	ditto	10			
H		21	10	ditto	7			
I	Wood	28	6	ditto	8			
K	Fir	18	10	ditto	10			

This Gudgeon broke from being a bad casting.

Broke, owing to their too great length, they were 11 inches long.

* The hollow shaft of C being 16 inches diameter, and 2 inches thick, broke in consequence of the gudgeon getting loose about March 7, 1808.

Observations on the first Table of Gudgeons.

154. Particular care should be taken that the axis of the gudgeon be exactly in a line with the axis of the shaft which it supports, otherwise the motion will be unequal, and at one part of the revolution the stress will be thrown to the point of the gudgeon; this would endanger its breaking, more particularly if very long, though otherwise sufficiently strong.* In the case H, we have an instance of a gudgeon breaking, from being made too long. In practice it is a good method to turn the gudgeons of a wooden shaft, after they are fixed in their places, a second time, in order to render them quite true.

155. From comparing the diameters of the gudgeons (column 6) with the cube root of the weight in cwts. of the wheel

* The possibility of such a cause of failure should be guarded against in proportioning the strength of a gudgeon. See Art. [A. 151.] (Ed.)

(col. 9,) it will be found that they approach one another. In other words, the cube root of the weight in cwts. is nearly equal to the diameter in inches. In the case C, the gudgeon broke, only from being a bad casting, although it is smaller in proportion to its weight, than most of the other cases. Since it was renewed of the same size, it has continued to support its work. When, therefore, we can ascertain the weight of a water-wheel, we have a very simple rule for finding the diameter which the gudgeon ought to have.

RULE I.

156. *The cube root of the weight of a water-wheel, in hundred weights, is nearly equal to the diameter in inches of a cast-iron gudgeon sufficiently strong to support such wheel.**

* In water-wheels, the stress is not proportional to the weight of the wheel. See note to Art. 151, where a rule on more correct principles is investigated. Also see note to Art. 152. (Ed.)

I say *nearly*, it being evidently most prudent to make the gudgeon a little more than less in diameter, and to make allowance for wearing.

EXAMPLE.

Suppose a water-wheel to weigh 12 tons, 0 cwt. 3 qrs. what ought to be the diameter of a cast-iron gudgeon, sufficiently strong to support the wheel?

$$12 \text{ tons} = 240 \text{ cwt. } 3 \text{ qrs.}$$

The cube root of $240.3 = 6.221$ *Answe.*
That is, the diameter of the gudgeon should not be less than $6\frac{2}{5}$ diameter. It ought to be rather more, to allow for wearing, &c. See B, in the first table of gudgeons.

157. As the weights of wooden water-wheels cannot be accurately known, without a good deal of calculation, it is desirable to have some more ready method for practical purposes.

The weights of overshot, or bucket wa-

ter-wheels, will be to one another nearly as their circumferences, or diameters and breadth.—I say *nearly*, because the arms will make large wheels heavy in rather a greater proportion. Hence the following rule is formed on the direct proportion of the sole and buckets adding one-half of the diameter increased in the duplicate ratio or square of the diameter.

RULE II.

For wooden water-wheels, multiply the diameter in feet by the width also in feet, to which add the square of half of the diameter. The cube root of the sum will be nearly equal to the diameter of the gudgeon in inches.

EXAMPLE.

Suppose a wooden water-wheel 12 feet diameter and 7 feet wide, (see E in Table II of Gudgeons.)

x 2

$$12 \times 7 = 84$$

$$\text{The square of } 6 = 36$$

$\sqrt{120} =$ the cube root of which $= 4.932424$; that is the gudgeon should not be less than about 5 inches diameter.

Explanation of Table II of Water Wheels.

158. All the columns, except No. 10, are the same as in Table I.* The column 10 shews the result by Rule II.

* In these tables the weight of the wheel B is different. I suppose the first table to be the correct one, but the difference is inconsiderable. (Ed.)

TABLE II. GUDGEONS OF WATER-WHEELS.

	Wheel made of	Diameter of wheel in feet.	Width in feet.	Kind.	Diameter of cast-iron gudgeon in inches.	Weight of wheel in tons.	Weight of wheel in cwt.	Diameter of gudgeon by calculating the weight.	Diameter of gudgeon of wooden water-wheels by Rule II.	Remarks.
A	Wood	24	12	Overshot	7	23	14	7.796974	7.559525	
B	Cast-iron	16	16	ditto	7	12	2	6.231678		
C	Cast-iron	16	8	ditto	6 $\frac{5}{8}$	16	10	6.910423		
D	Cast-iron wheel and buckets	32	4 $\frac{1}{2}$	ditto	9	20		7.368063		
E	Wood	12	7	ditto	6				4.932424	
F	Wood	32	11	ditto	10	5	14	4.848808	8.471647	
G	Wood	21	10	ditto	7				6.839903	
H	Wood	28	6	ditto	8				7.140037	
I	Wood	16	9	ditto	6	10	11	5.953341	5.924991	
K	Wood	18	10	ditto	10				6.390676	

SECTION III.

Of Cast-iron Gudgeons for various Purposes.

159. Taking it for granted that the cube root of the weight of a water-wheel, in hundred weights, is nearly equal to the diameter in inches of a cast-iron gudgeon, sufficiently strong to support such wheel, the following table of the diameters of gudgeons, and the weights which they may be supposed to sustain, is formed. It may be of use for finding the diameters of journals in all cases of stress arising from lateral pressure, such as grindstones, intermediate spindles, &c. where the pressure can be ascertained, as well as water wheels.*

Explanation of the Table of Cast-iron Gudgeons.

160. Column 1 contains the diameter in inches, from 1 to 11 inches.

* See Art. [B. 151.]

Column 2 contains the cube of that diameter, or the hundred weights which the gudgeon may sustain.

N. B. I have already remarked, (Art. 156,) that it would be most prudent to make the gudgeon a little more in diameter than the cube root of the hundred weights.

Table of Cast-iron Gudgeons.

Diameter in inches.	Cube of Diameter, or cwts. which the gud- geon may sustain.	Diameter in inches.	Cube of Diameter, or cwts. which the gud- geon may sustain.
1.	1.	6.25	244.140625
1.25	1.953125	6.5	274.625
1.5	3.375	6.75	307.546875
1.75	5.359375	7.	343.
2.	8.	7.25	381.078125
2.25	11.400625	7.5	421.875
2.5	15.625	7.75	465.483375
2.75	20.796875	8.	512.
3.	27.	8.25	561.515625
3.25	34.328125	8.5	614.125
3.5	42.875	8.75	669.921875
3.75	52.734375	9.	729.
4.	64.	9.25	791.453125
4.25	76.765625	9.5	875.375
4.5	91.125	9.75	926.859375
4.75	107.171875	10.	1000.
5.	125.	10.25	1076.890625
5.25	144.703125	10.5	1157.625
5.5	166.375	10.75	1242.296875
5.75	190.109375	11.	1452.
6.	216.		

Use of the Table.

EXAMPLE I.

161. Suppose a cast-iron gudgeon of $5\frac{1}{2}$ inches diameter, what weight of a water-wheel would it be capable of sustaining?

In the first column find 5.75. Opposite to which will be found 190.109375 hundred weights, which is rather more than $9\frac{1}{2}$ tons, which is the answer. But in practice, it would perhaps be prudent not to load this gudgeon with more than nine tons.

EXAMPLE II.

Suppose a grindstone weighing $15\frac{1}{2}$ hundred weights, required the size of a gudgeon sufficient to sustain this weight. Look in the second column for the nearest weight to that given, which will be found to be 15.625, opposite to which, in the first column, is $2\frac{1}{2}$ inches, the diameter of the gudgeon required.

But in practice the spindles of grindstones are commonly of wrought iron, which metal I shall presently consider as applied to gudgeons.

SECTION IV.

Of Malleable or Wrought-iron Gudgeons.

162. Professor Robison states,* that the cohesive force of a square inch of cast iron is from 40,000 to 60,000 lb. wrought iron from 60,000 to 90,000 lb.

In the year 1795, I had occasion to substitute cast-iron gudgeons for those of wrought iron, and made some experiments on those metals, from which I drew the following inference: *that gudgeons of the same size, of cast and of wrought iron in practice, are capable, at a medium, of sustaining weights without flexure, in the proportion of 9 to 14.†*

* Encyclopedia Britannica, article Strength of Materials, 40.

† According to my experiments, the stiffness of good cast-iron is to that of good English malleable iron as

Taking it for granted that this proportion is near the truth, we may find the diameter which any wrought-iron gudgeon ought to have when its lateral pressure is given, in the following manner:

1 is to 1.3 nearly. (Essay on Cast-Iron, Art. Iron.) But the stiffness of malleable iron is much increased by hammering, &c.; and in my trials some pains were taken to obtain the resistance unhammered, which most probably causes the difference.

It is further necessary to observe, that in calculating the strength of gudgeons, the resistance to permanent alteration is the proper measure, because they are too short to admit of sensible flexure; and the strain which produces permanent alteration in malleable iron is only 1.12 times that producing a like alteration in cast iron.

Hence, the diameter of a malleable iron gudgeon should be 0.963 times that of a cast-iron one to bear the same stress. For if a be the diameter of the cast-iron gudgeon, and b the diameter of the wrought-iron one; then $a^3 \times 1 = b^3 \times 1.12$ when their strengths are equal; consequently $\frac{a^3}{1.12} = b^3$; or $\frac{a}{(1.12)^{\frac{1}{3}}} = b$; but $\frac{a}{(1.12)^{\frac{1}{3}}} = 0.963 a = b$.

The proportion given by our author is about $0.863 a = b$, but his proportion is made from the relative stiffness, while the rule applies to the strength, and therefore it is not correct. (Ed.)

163. 1. Find the diameter which a cast-iron gudgeon should have to sustain the given pressure, then say, as 14 is to the cube of the diameter of the cast-iron gudgeon, so is 9 to the cube of the diameter of the wrought-iron gudgeon.

2. The root of this last number gives the diameter required of the wrought-iron gudgeon.

EXAMPLE.

Suppose the lateral pressure to be 125 hundred weights, the cube root of which is 5, the diameter in inches of the cast-iron gudgeon : then say,

As 14

Is to 125

So is 9

To . . . 80.357

The cube root of which is 4.30887.*

* To find the proportion by the preceding note, multiply the diameter of the cast-iron gudgeon by 0.963. Thus 5 inches $\times$ 0.963 = 4.815 inches, the diameter for a wrought-iron one. (Ed.)

Upon this principle the following table is calculated to shew the proportionate diameters of cast-iron and wrought-iron gudgeons.

Explanation of the Table of Cast and Wrought-iron Gudgeons.

164. Column 1 and 2 are the same as those in the table of cast-iron gudgeons.

Column 3 contains numbers in the proportion of 9 to 14 less than those of column 2.

Column 4 contains the cube root of column 3, or the diameters of wrought-iron gudgeons, having the same strength as those of cast-iron in column 1.

Table of Cast and Wrought Iron Gudgeons.

4	2	3	4
Diameter of gudgeons in inches.	Cube of Diameter of cast-iron gudgeons, or the cwts. which the gudgeons may sustain.	Cube of Diameter of wrought-iron gudgeons.	Diameter of wrought-iron gudgeons, in inches and parts.
1.	1.	.6428571	.863
1.25	1.953125	1.2555803	1.063340
1.5	3.375	2.1696427	1.259921
1.75	5.359375	3.4453125	1.514825
2.	8.	5.1428571	1.709976
2.25	11.400625	7.3289732	1.912933*
2.5	15.625	10.0446428	2.154435
2.75	20.796875	13.3694196	2.351335
3.	27.	17.3571428	2.571282
3.25	34.328125	22.0670803	2.802039
3.5	42.875	27.5625	3.018294†
3.75	52.734375	33.9006696	3.239612
4.	64.	41.1428571	3.448217
4.25	76.765625	49.3493303	3.659306
4.5	91.125	58.5803571	3.881936
4.75	107.171875	68.896	4.101566
5.	125.	80.357	4.308870
5.25	144.763125	93.023	4.530655
5.5	166.375	106.955	4.747459
5.75	190.109375	122.213	4.959675
6.	216.	138.857	5.180101
6.25	244.140625	156.948	5.394690
6.5	274.625	176.545	5.609376
6.75	307.546875	197.709	5.828476

* Dr. Brewster says, "A gudgeon 2 inches diameter, will sustain 3239lb. avoirdupois." Brewster's Edition of Ferguson's Lectures, vol. i. p. 157.

† The wrought-iron grindstone spindle used by Mr. Southern in his experiments on Friction, was $3\frac{1}{8}$ diameter, and carried a stone weighing 3700lb. See Phil. Mag. Vol. xvii, 123.

Table of Cast and Wrought Iron Gudgeons, continued.

1	2	3	4
Diameter of gudgeons in inches.	Cube of Diameter of cast-iron gudgeons, or the cwts. which the gudgeons may sustain.	Cube of Diameter of wrought-iron gudgeons.	Diameter of wrought-iron gudgeons in inches and parts.
7.	343.	220.500	6.041377
7.25	381.078125	244.979	6.257324
7.5	421.875	271.205	6.471274
7.75	465.484375	299.240	6.686882
8.	512.	329.143	6.903436
8.25	561.515625	360.975	7.120367
8.5	614.125	394.795	7.337234
8.75	669.921875	430.664	7.553688
9.	729.	468.643	7.769462
9.25	791.453125	508.791	7.984344
9.5	875.375	562.741	8.257263
9.75	926.859375	595.837	8.415541
10.	1000.	642.857	8.631103
10.25	1076.890625	692.287	8.845085
10.5	1157.625	744.187	9.061309
10.75	1242.296875	798.619	9.279308
11.	1452.	933.428	9.771484
<i>Use of the Table.</i>			
EXAMPLE.			

165. To find the diameter of a wrought-iron gudgeon of the same strength with one of cast-iron of 3 inches diameter. Look on the 1st column for 3, and on the same line in the 4th column will be found

2.571282, that is, a little more than $2\frac{1}{2}$ inches, the diameter required of the wrought-iron gudgeon.

The numbers in the 3rd column, being the cube of those in the 4th, another use may be made of this part of the table. For, supposing the 4th column to represent cast-iron gudgeons, then the 3d column will represent the hundred weights which cast-iron gudgeons of those diameters should sustain.

Before proceeding to consider the bodies of Shafts, subject to *lateral stress*, I shall inquire into the Strength of *Journals* of Shafts subject to torsion.

CHAP. III.

SECTION I.

*Of the Strength of Journals, when the Stress arises from Torsion or Twisting, in addition to lateral Stress.**

166. In my inquiry into the strength and durability of the teeth of wheels, I have used what is called the *horses' power*, as a measure for the strain. I beg leave to refer the reader to what I have there said, (Art. 79—85.) in explanation of that term which I shall use here, in measuring the strain brought on shafts by torsion or twisting.

* When a shaft has a support between the points where the power and resistance are applied, the part of the shaft which revolves on this support is called a *Journal*. (Ed.)

In this case of torsion, as well as that of lateral pressure, the law of proportionate strength *is as the cubes of the diameters*.*

It may be proper here to remark, that what I have to say respecting journals, relates to those of cast iron, for although wrought iron will bear more lateral stress, as we have seen, (Art. 162,) yet it is a fact, perhaps, not generally known, that wrought iron will not resist torsion equal to cast iron.†

In some cases a journal has not only torsion to resist, but also to carry a very heavy fly wheel, and it is prudent in such cases not merely to make an allowance for the weight properly balanced, but also for any inaccuracy which may occasion swagging, which greatly adds to the stress: but others have hardly any other resistance but what arises from torsion.

* See Gregory's Mechanics, vol. i. art. 191.

This reference is to a statement that the strength is in this proportion, but it is not demonstrated. (Ed.)

† The author seems to be under a mistake in regard to this fact. (Ed.)

It is further observable that the value for 10 horses in the smaller engines is much less than in the larger. For this difference what I have said respecting the weight of the fly in a great measure accounts: and not only is the heavier fly to be considered, but also the greater danger of accidents from a large fly than from one that is of a smaller diameter.

SECTION II.

Of proportioning Journals to the Stress which they have to sustain.

167. The stress any journal has to sustain being as the horses' power to which the resistance is equal *directly*, and the number of revolutions which the shaft makes *inversely*, it follows: That a resistance for example of 32 horses' power on a journal making 50 revolutions per minute, has the very same stress with a resistance of 16 horses' power on another journal making 25 revolutions per minute.

50 divided by 32 is equal to 25 divided by 16, each of which gives a quotient of 0.64. Therefore in all cases when the horses' power divided by the revolutions per minute produces the same quotient the stress is the same.

Thus a resistance equal to 50 horses' power making 50 revolutions per minute produces the very same stress as 10 horses' power making 10 revolutions per minute.

Having therefore fixed on any journal which has been found sufficiently strong, we may make any other to have the same strength in proportion to the resistance which it has to overcome in the following manner.

168. RULE. If it so happen that the horses' power, and the revolutions per minute be the *same number*. For instance, 50 horses' power making 50 revolutions, $50 \div 50 = 1$; then the cube of the diameter of the journal will be a multiplier, by which to find the cube of the diameter of the required journal.

But in case the horses' power and the revolutions per minute are *different numbers*, then you must suppose them both the same, and calculate (as in Ex. II.) what, in that case, would be the proportionate diameter of the journal—The cube of this diameter will be a multiplier, the same as mentioned above.

Having found the multiplier, to find the diameter of the required journal.

Divide the horses' power by the revolutions per minute.

Multiply the quotient by the *multiplier*, the cube root of the product will give the diameter of the journal required.”*

EXAMPLE I.

To find the multiplier from a journal $7\frac{1}{2}$ inches diameter, where there is an engine of 50 horses' power turning a shaft, at the rate of 50 revolutions per minute.

* See Art. 171.

Divide the power by the revolutions, that is 50 divided by 50 is equal to 1; the diameter of the Journal is $7\frac{1}{2}$ inches; the cube of this is 420, which multiplied by 1 produces 420.

In this case it happens that the horses' power, and the revolutions per minute, are *the same number*, therefore we with little trouble find the multiplier.

EXAMPLE II.

From a journal of 4 inches diameter, where the horses' power is 12, and the revolutions per minute 48; to find the multiplier.

Now let us suppose both numbers the same, that is, 12 horses' power, and 12 revolutions.

Here it is evident that there will be four times the stress brought on the journal. Its actual diameter was 4 inches.

Then the cube of 4 is 64,

64 multiplied by 4 is 256 inches,

The cube root of which is 6.35,

which is the diameter which the journal ought to have, to be in proportion to the velocity.

The cube of 6.35 is 26, which is the multiplier required.

It is to be observed, that in the latter case a much smaller steam engine is employed than in the former, and, therefore, for reasons already given (Art. 166) has less stress brought upon the journal. This accounts for the multiplier being less.

I shall now give an example of the application of a *multiplier*; let us take that found in the case Example I. viz. 420, and see what size of the journal it would give in the case Example II. which is an engine of 12 horses' power and journal making 48 revolutions per minute.

12 divided by 48, equal to $\cdot 25$, then multiplied by 420, gives a quotient of 105, that is, the strength of the journals must be as 420 to 105; but the cube root of 420 is $7\frac{1}{2}$, and the cube root of 105 is $4\frac{3}{4}$, which points out that the journals $7\frac{1}{2}$ and $4\frac{3}{4}$ are proportioned to their respective en-

gines. In like manner the following table is calculated; the multiplier being 420.

169. *Description of the Table of Journals, proportionate to D, having 420 as a multiplier.*

Column 1	contains letters to distinguish the cases in which D is the same as Ex. I. (Art. 168,) and E, Ex. II. of the same Art.
----------	--

Column 2	the horses' power.
----------	--------------------

Column 3	the revolutions of the journal per minute.
----------	--

Column 4	the product of the horses' power divided by the revolutions of the shaft.
----------	---

Column 5	contains the proportionate strain on each journal, represented in whole numbers, which are found by multiplying the product in column 4, by 420, as a multiplier.
----------	---

Column 6	diameters of journals, as really executed in several steam engines.
----------	---

TABLE OF JOURNALS

Proportionate to D, having 420 as a multiplier.

1	2	3	4	5	6
	Numbers of horses' power.	Revolutions of Journal p. min.	Product of power divided by the rev. of the Journal.	Proportionate strain on Journal.	Diameters of Journals from observation.
A	32	58	0.55	231	$7\frac{1}{2}$
B	32	19	1.67	701	$9\frac{1}{2}$
D	50	50	1.0	420	$7\frac{1}{2}$
E	12	48	0.25	105	4
F	9	55	0.16	67	4

Observations.

170. I have already observed (Art. 166,) that, besides torsion, the journals of fly wheel shafts have considerable lateral, and other stress, arising from the weight and swagging of their fly wheels, and therefore, they ought to be made stronger than shafts, in other situations. The multiplier 420, therefore, which we have used in the table, would give diameters too great, for some other parts of machinery. A journal, for

instance, subject to torsion, immediately connected with a water wheel, has from the weight of the wheel and other causes, considerable lateral stress; but not so much as that of a steam engine. The journal may therefore, be considerably smaller than would be required for a steam engine fly-wheel shaft, subject to the same degree of torsion.*

Again, a secondary shaft driven from a steam engine, a water wheel, or horse gin, by means of wheels, has in general very little lateral stress, compared with the two cases just stated; and may therefore have a journal, smaller than either, when the degree of torsion is the same.

171. For these reasons the three following multipliers will probably approach near the truth; that is, for journals of steam engine fly-wheel shafts (where the power is moderate.) 400

* It seems a better method to use a mode of calculation which includes the effect of lateral stress; see Art. [B. 171.] (Ed.)

Journals in immediate connection
with water-wheels,* or other
heavy work. 200

Journals for the ordinary kind of
internal millwork 100

EXAMPLE.

Suppose B, in the table, (Art. 169,) 1.67
multiplied by 400, is 668, the cube root of
which is 8.7416 inches, diameter of journal.

[A. 171.] When the resistance of a
journal is equal to the twisting stress, the
strain not being sufficient to produce per-
manent derangement in the material, the
cube of the diameter of the journal will be
equal to 3.78 times the number of horses'
power divided by the number of revolu-
tions in a second.†

* The reader will please to observe, that, when I use
the word journal here, I suppose it subject to torsion.
Where there is lateral pressure only, and no torsion, I
would use the word gudgeon.

† The reason of this rule will be given in treating of
the resistance of shafts.

But some allowance must be made for wear, and if this be made so that the journal shall have sufficient strength when it is worn down one-sixth of its diameter, the number 3.78 should be made 6.01; or with sufficient accuracy 6.

If the number of revolutions in a minute be employed instead of those in a second, the constant multiplier, 6, must be multiplied by 60; and therefore the constant multiplier will become 360; and a less number ought not in any case to be employed, because there will always be some lateral stress in addition to the twisting stress. The resistance of a journal, or its diameter as regards the twisting strain, may be always calculated by the following rules.

RULE. If N be the number of revolutions in a minute, and d the diameter of the journal in inches, then $\frac{N \cdot d^3}{360}$ = the number of horses' power the journal is sufficient to resist.

RULE. If N be the number of revolu-

tions per minute; and H the number of horses' power moving the train of machinery, then $7.12 \times \left(\frac{H}{N}\right)^{\frac{1}{3}} = d$, the diameter of the journal in inches.

EXAMPLE.

Let it be required to find the diameter of a journal for case B in the table of journals, Art. 169; then we have H equal 32 horses, and the number of revolutions N equal 19; therefore $\frac{32}{19} = 1.68421$. The cube root of 1.68421 is found by the table Art. 373, to be 1.19; and $7.12 \times 1.19 = 8.4728$ inches, the diameter of the journal, or nearly $8\frac{1}{2}$ inches.

[B. 171.] But the effect of lateral stress ought always to be considered, and we have found the strength of a gudgeon to be $0.6 (W l)^{\frac{1}{3}} = d$; where the stress is wholly lateral, (Art. B. 151.) Or, $0.216 W l = d^3$. And the strength of a journal, where the stress is altogether twisting to be $\frac{360 H}{N}$

$= d^3$; therefore, since journals bear both kinds of stress, we have as a general

RULE. $\left(0.216 \ W l + \frac{360 H}{N}\right)^{\frac{1}{3}} = d$, the diameter in inches. Where W is the lateral stress upon the journal in cwts. l the length of the journal in inches, H the number of horses' power moving the train of machinery, and N the number of revolutions of the journal per minute. (Ed.)

SECTION III.

172. When the diameter of a journal and its revolutions per minute, are given, in order to find the horses' power to which it is equal ; we must invert the preceding operation, and convert the *multiplier* into a *divisor*.

RULE.

Cube the diameter of the journal, divide the cube by the *divisor*. The quotient multiplied by the revolutions per minute, gives the horses' power, to which the journal is equal.*

* See additions to the preceding article.

EXAMPLE I. *F in the table.*

Suppose the journal of a steam engine to be 4 inches diameter, making 55 revolutions per minute, then we shall use 400 as a divisor.

The cube of 4 is 64, divided by 400 equal to 0.16, then this quotient (0.16) multiplied by 55, gives 8.8 the horses' power, to which the journal is equal.

EXAMPLE II.

Suppose again, the same size of a journal, connected with heavy machinery, then we must use 200 as a divisor.

The cube of 4 = $64 \div 200 = .32 \times 55 = 17.6$ horses' power.

EXAMPLE III.

We shall take the same journal for internal work, of the ordinary kind, and use 100 as a divisor.

The cube of 4 = $64 \div 100 = .64 \times 55 = 35.2$ horses' power.

CHAP: IV.

SECTION I.

On the Bodies of Shafts.

173. From what is stated in the preceding part of this Essay, I hope the Millwright will be enabled to approach sufficiently near to the truth, for all practical purposes in proportioning gudgeons and journals to the stress which they have to sustain. Taking this for granted, it may be proper for us next to consider what relates to the *bodies* of shafts, or those parts which lie between the gudgeons or journals. In this part of our inquiry, we may derive assistance from the principles which have been applied by writers on mechanics, to the stress of timber and other materials.* The generality of writers on mechanics, however, as Dr. Young justly observes,

* Few operative mechanics have a distinct notion of the difference between *strength* and *stiffness*.

(vol. i. page 136,) have confined their attention to *strength* (resistance to fracture) alone, although there be other very important properties, which required their consideration. The most usual as well as the most important effect, produced by the application of force is *flexure*: (p. 138, *ibid.*) *stiffness* therefore as well as *strength*, ought to be considered in determining the form, as well as the quantity of materials, for any mechanical purpose, more particularly that of a shaft in millwork, which in theory may be considered as an *inflexible* straight line. I beg the practical reader would attend to the distinction between *stiffness* and *strength*. Stiffness is that property, which resists *flexure* or *bending*. Strength that which resists fracture or breaking. The consideration of their limits, may make this plainer. The limit of *stiffness* is *flexure*; the limit of *strength* is *fracture*. The *stiffness* of a beam follows laws very different from those which determine its *strength*; these laws I shall presently consider, and endeavour to shew their appli-

cation to practice, with regard to some cases of shafts.—But although those laws may throw considerable light on the subject, yet it must be confessed, that there are many cases in practice, in which it is very difficult, if not impossible, to apply them ; for it is very often difficult to estimate what may be the amount of the lateral pressure on shafts, arising not only from their own weight, and that of the wheels, upon them, but also from the thrust, proceeding from the action of the toothed wheels, and other extraneous causes. In cases of this nature, where calculation fails, much must be done, by what Mr. Smeaton calls *feeling*,* which will direct the experienced millwright to make a due allowance for whatever accidental strain may be likely to occur.

I shall now proceed to state and apply some of the laws, respecting stiffness and strength, with regard to force, applied transversely.

Smeaton's Account of Eddystone Lighthouse, p. 136.

Of lateral Stiffness and lateral Strength.

174. “The *stiffness* of any substance, is measured by the force required to cause it to recede, through a given small space, in the direction of the force,” (Young’s Nat. Phil. vol. i. p. 139.) Its *transverse strength* is measured by the pressure required to produce its fracture, or, in other words, to break it.

PROPOSITION II.

175. *Any beams of equal length, have their lateral Stiffness, [to bear a load at any point in the length,] as the breadth and cube of the depth* (Young’s Nat. Phil. vol. i. p. 139, or ii. art. 333,) *and have their lateral Strength, as the breadth and square of the depth,** (Gregory, vol. i. art. 169, cor. 1. Emerson, prop. 67.)

* That is, as the cube of the side of a square beam, and in general the lateral strength of any beams whose sections are similar, as the cube of the similar sides or diameters of the sections.

Thus, if a square beam measure twice as much, on the side, as another of equal length, it would be sixteen times as stiff. In other words, it will sustain sixteen times the weight, without bending.

But, if a square beam be twice as much on the side as another, both being the same length, it will be only eight times stronger.

Hence we see, that when beams or shafts are of equal lengths, their *stiffness*, by any increase of thickness, increases in a higher proportion than their *strength*.

EXAMPLE I.

If a beam or shaft be four inches square throughout, and another five inches, both of equal lengths; what is their comparative *stiffness*?

The cube of 4, is 64,

$$64 \times 4 = 256,$$

The cube of 5, is 125,

$$125 \times 5 = 625,$$

That is, the shaft of five inches is nearly

nearly two and a half times stiffer than that of four; in other words, it would require nearly two and a half times the weight to bend it.

EXAMPLE II.

If a beam or shaft be four inches square throughout, and another five inches, both of equal lengths; what is their comparative strength?

The cube of 4, is 64,

The cube of 5, is 125,

That is, the five inches shaft is nearly twice as strong as that of four inches; in other words, it would require nearly double the weight to break it.

PROPOSITION III.

176. *Any beams of different lengths, have their stiffness [to bear a load at any point in the length] directly as the breadth and the cube of the depth, and inversely as the cube of the length, (Young's Nat. Phil. ii. art. 333,) and have their strength directly*

*as the breadth, and as the square of the depth, and inversely as the length**, (Young, vol ii. art. 335.)

Thus, if a beam be twice as long as another, of the same breadth and depth, it will have only one eighth of the stiffness, while it will have one half of the strength. Hence the *stiffness* of shafts or beams by any increase of their length, decrease in a much higher proportion than that of their *strength*.

* This is not strictly true in practice, for "some experiments appear to shew, that the strength is diminished, in a proportion somewhat greater than that in which the length is increased." (Young's Nat. Phil. vol. ii. p. 147.)

The variation is caused by the increase of strain which takes place when the flexure is considerable, (see Elementary principles of Carpentry, Art. 18,) and some decrease of cohesive power when the natural arrangement of the particles of a body are disturbed more than in a certain degree; but these causes are insensible in a practical point of view, because we can never allow the stress to produce so much flexure, nor the strain to be so near to fracture, as to make it necessary to allow for such circumstances. (Ed.)

EXAMPLE I.

Suppose a beam or shaft, four feet long and four inches square throughout, and another eight feet long and seven inches square; what is their comparative stiffness?

The cube of 4 feet, is 64,

The cube of 8 feet, is 512,

512 divided by 64, is equal to 8, that is, when we double the length, we decrease the stiffness eight times.

The cube of 4 inches is 64, which multiplied by 4 is equal to 256, a number representing the stiffness of the four inch shaft.

The cube of 7 inches is 343, multiplied by 7 is equal to 2401, divided by 8 is equal to 300.1, then as 256 is to 300.1, so is the stiffness of the shaft of four inches to that of a shaft of seven inches.

EXAMPLE II.

Suppose a beam or shaft, four feet long and three inches square, and another eight

feet long and seven inches square, what is their comparative strength?

The cube of 4 is 64, which represents the strength of the four inch shaft.

The cube of 7 is 343; but the shaft being of double length, we must halve this sum, to find the number representing its strength, viz. $343 \div 2 = 171.5$ divided by 100, that is, as sixty four is to a hundred and seventy-one and a half, so is the strength of the short shaft to that of the long one. Thus the shaft of seven inches, eight feet long, has nearly two and six tenth times the strength, of the four inch four feet long.

PROPOSITION IV.

177. *Supposing a tube, indefinitely thin, to be expanded into a similar tube of a greater diameter, but of equal lengths, the quantity of matter remaining the same, the STIFFNESS will be increased, in the ratio**

* Ratio, that is proportion.

*of the square of the diameter, and the STRENGTH in the ratio of the diameter.**

Thus, if the one tube be double the diameter of the other, it will have four times its *stiffness*, but only double the strength.

Hence, hollow cylinders of equal lengths, by any increase of diameter, increase in *stiffness*, in a much higher proportion than in *strength*.

EXAMPLE I.

Suppose two thin narrow cylindrical cast-

* This proposition is taken from Dr. Young's Nat. Phil. vol. ii. art. 339, where it is followed by this essential limitation. "When a beam of finite thickness is made hollow, retaining the same quantity of matter, the strength is increased in a ratio somewhat greater than that of the diameter, because the tension of the internal fibres at the instant of breaking is increased." Dr. Young has given the correct rule for estimating the strength and stiffness of a hollow cylinder, at p. 84, (Nat. Phil. vol. ii.) "The strength of a tube may be found by deducting from the strength of the whole cylinder that of the part removed, reduced in the ratio of the diameters." And observes, that "the strength is in this case in the same ratio as the stiffness." (Ed.)

iron shafts, of equal lengths and weights, the one of one foot diameter, and the other three feet diameter, required their comparative *stiffness*?

The square of one is one,

The square of three is nine,
that is, the shaft of three feet diameter, is nine times stiffer than that of one foot.

EXAMPLE II.

Suppose the same shafts, as in example first, required their comparative strength?

Diameter one foot,

Diameter three feet,

that is, the three feet shaft is just three times stronger, than that of one foot diameter.

178. In these examples, we have supposed the weight of the shafts equal, that is the area of their ends to be equal, but the strength of any of them would be increased in proportion to their weight, or the

areas of their ends and diameters, conjointly, (Gregory, vol. i. art. 172, cor. 3.)

Thus suppose two shafts of equal length and diameter, the one double the weight of the other, it will be double the strength.*

179. Professor Robison justly observes, "That this property of hollow tubes is accompanied also with greater stiffness, and the superiority in strength and stiffness is so much the greater, as the surrounding shell is thinner in proportion to its diameter. Here we see the admirable wisdom of the Author of nature in forming the bones of animal limbs hollow. The bones of the arms and legs have to perform the office of levers, and are thus opposed to very great transverse strains. By this form they become incomparably stronger and stiffer, and give more room for the insertion of muscles, while they are lighter and therefore more agile; and the same

* See note to Art. 177. (Ed.)

wisdom has made use of this hollow for other valuable purposes of the animal economy. In like manner the quills in the wings of birds acquire by their thinness the very great strength which is necessary, while they are so light as to give sufficient buoyancy to the animal, in the rare medium in which it must live and fly about. The stalks of many plants, such as all the grasses, and many reeds, are in like manner hollow, and thus possess an extraordinary strength." (Ency. Brit. article Strength.)

Long before this eminent philosopher, the celebrated Galileo made similar observations, and goes on to say that "if a wheat straw, which supports an ear that is heavier than the whole stalk, were made of the same quantity of matter but solid, it would bend or break with far greater ease than it now does. And with the same reason art has observed and experience confirmed, that a hollow cane or tube of wood or metal, is much stronger and more firm than if, while it continued of the same weight

and length, it were solid, as it would then of consequence be not so thick.

It may be proper now to consider the effects called *stress*, which are produced on beams or shafts lying horizontally by weights or pressures brought on various parts of them.

SECTION II.

Of Lateral Stress.

180. The *stress* or *strain** are terms used to express the force which is excited in any body tending to break it. The meaning of the term *stress* may perhaps be more clearly understood by contrasting it with the term *strength*.

Strength, as we have already observed, is the property which *resists fracture*.

Stress is that which has the tendency to produce *fracture*; and *lateral stress* is that

* Strain is the effect of stress: it is the derangement from the natural state which is caused by stress. (Ed.)

particular application of it, which has the tendency to break a body across.

PROPOSITION V.

181. *The stress on a beam arising from one weight hung upon it, is proportional to the rectangle of the parts of the beam, and is greatest when the load is laid on the middle of the beam.* (Ency. Brit. art. Roof, § 19.)

What is meant by the expression *rectangle of the parts*, is the product of parts multiplied into each other. Thus for example: if a beam be ten feet long, and the weight hung two feet from one end, the parts are 2 and 8, which multiplied together, would be equal to 16; but supposing the weight were hung in the middle, the parts are 5 and 5, which multiplied together would produce 25.

182. The ends of beams having the whole weight to support; the end which is nearest the weight, has to support the greatest proportion of it, in the inverse pro-

portion of the distance of the weight from the end. This will be easily understood from the properties of the lever. For, suppose the beam instead of being supported by two props or walls, as in Fig. 9, No. 1, to be hung from each end by a rope, as in Fig. 9, No. 2, it is plain that the beam would receive the same support, and suffer the same stress, as if lying on props or walls; now suppose the weights A and B, to balance the weight W, then A and B, taken together, must be equal to W, but A must be greater than B in proportion as W is near to it.

183. Hence when any beams or shafts are loaded exactly in the middle, each of the ends of the beams or gudgeons of the shafts has half the weight to support, and when the weight is nearer one end, the end or gudgeon to which it is nearest, has the stress in the inverse proportion of the distance. In this last case, therefore, the one gudgeon might be smaller than the other.

184. "We may always consider the weight which is uniformly diffused over any part of a beam as united in the middle of that part, and if the load is not uniformly diffused, we may suppose it united at its centre of gravity." (Ency. Brit. article Roof, § 20.)*

185. It is evidently of importance that a beam or shaft should in every part be able to resist the strain excited in that part. "It should therefore be equally strong, because the piece will nevertheless break where it is not stronger throughout, and it is useless to make it stronger (relatively to its strain) in any part, or it will nevertheless equally fail in the part that is too weak." (Ibid.)

* When the weight is uniformly diffused, the stress is the greatest at the middle of the length, and is equal to half the weight collected in the middle. But the flexure in the middle produced by a weight which is uniformly diffused, is the same as when five eighths of the load is collected in the middle of the length. (See art. Carpentry, Supplement to Encyclopædia Brit. 1817. Prop. F. or Barlow's Essay on the Strength of Timber, p. 117.)

186. From what we have said respecting lateral stress, it is evident that when a beam lying between two props is loaded at some intermediate part, that part has to sustain more stress than the rest. In order to resist this strain, therefore, and to render the beam equally strong throughout, it should have its section enlarged at the place of greatest stress, and hence shafts subject to lateral stress, should swell in the middle, and it will be found that when each section is made proportional to the stress it has to sustain, that the sides of the shaft will form curves.

187. When the transverse sections of a beam are all similar, such as circles, squares, or polygons, and the weight is laid on one place, in order to make it equally strong throughout its length, the curve of the sides of the beam becomes what mathematicians call a cubical parabola. (En. Brit. Strength of Materials, 87.) But when the weight is uniformly diffused all over the beam, the sides of the beam must

be a different parabolic curve called a semi-cubical parabola.*

We come now to examine some of the laws respecting twisting or torsion.

SECTION III.

Of Torsion.

PROPOSITION VI.

188. *In general the strength of a cylinder or solid axle by which it resists being wrenched asunder by twisting is as the cube of its diameter.* (Ency. Brit. art. Strength of Materials, 123.)

Thus if a solid cylinder be double the diameter of another, it would require eight times the force to wrench it asunder.

Of Hollow Axles.

189. Hollow axles are stronger to resist twisting than solid ones containing the

* "The parabola is a conic section, arising from a cone being cut by a plane parallel to one of its sides, or parallel to a plane that touches one side of the cone."

same quantity of matter. For if a hole be bored out of an axle of half its diameter, this reduces its *weight one fourth* (because circles are to one another as the squares of their diameters,) but the strength of solid cylinders being as the cubes of their diameters, the part taken out by boring had only the eighth part of the strength of the whole cylinder, and therefore when taken out would reduce the *strength* of the whole *one eighth*.

Thus let the external diameter of the hollow axle be five inches, and that of the hollow part of it four inches, then the diameter of another cylinder made solid, having the same quantity of metal with the tube, is three inches.

For 5 multiplied by 5 is equal to 25

4 multiplied by 4 is equal to 16
difference 9

The square root of 9 is 3. The strength of the solid cylinder of five inches diameter may be expressed by the cube of 5, or 125. Of this the internal part four inches diameter exerts 64, that is, the cube of 4;

therefore the strength of the tube is 64, subtracted from 125, is equal to 61, but the strength of the solid axle of the same quantity of matter, and three inches diameter is expressed by the cube of 3 or 27, which is not half of that of the tube. (En. Brit. art. Strength of Materials, 124.)*

190. The superiority of strength of hollow tubes over solid cylinders is much greater in resisting torsion than transverse or lateral stress. We have seen above, that the strength to resist torsion of the tube was to that of the cylinder as sixty-one is to twenty-seven; but Professor Robison estimates, that their strength to resist transverse strain is only as sixty-one is to thirty-two and a half nearly—and if we calculate according to Dr. Gregory's corollary, vol. i. page 109, (see Art. 178 of this Essay,) the result will be still more in

* These calculations are founded on the erroneous supposition that the tension is equal in every part of the section; and consequently they are widely distant from the truth. (See note to Art. 190.) (Ed.

favour of strength to resist torsion : for by the last mode of calculation the tube would be to the cylinder only as forty-five is to thirty-six ; but the Professor's mode of calculation, though less simple, is probably more accurate than that above alluded to.*

* It has been shewn that the resistance of a cylinder to torsion is $124.8 d^3 = R W$; where W is the stress in lbs. and R the leverage in feet it acts with ; d being the diameter of the cylinder in inches. (Essay on Cast-Iron, Art. 227.) And when the straining force is considered to act at the surface of the shaft $\frac{d}{2} = 12 R$; and therefore in this case $20.8 d^2 = W$.

By the same reasoning it may be proved that, in a hollow cylinder, where D is the exterior diameter, and $n D$ the diameter of the hollow part, $124.8 D^3 (1 - n^4) = R W$; when the force W , acts with the leverage R in feet ; but when the force is applied at the surface of the shaft, $20.8 D^2 (1 - n^4) = W$.

Hence when the strain is at the surface of the shaft, the resistance of a solid cylinder is to that of a hollow one as $d^2 : D^2 (1 - n^4)$; and taking the same example, which our author has quoted from Professor Robison, we have $d = 3$, $D = 5$, and $n D = 4$, or $n = .8$, the ratio is $9 : 25 (1 - .4096)$, or as $9 : 14.76$, when the stress is reduced to the surface of the shaft or cylinder.

But the ratio is $d^3 : D^3 (1 - n^4)$ when the leverage is

191. Professor Robison mentions (in En. Brit. art. Strength of Materials, § 128) that “when the matter of the axle is of the most simple texture, such as that of metals, we do not conceive that the length of the axle has any influence on the fracture. It is otherwise if it be a fibrous texture, like timber; the fibres are bent before breaking, being twisted into spirals like a corkscrew. The length of the axle has somewhat of the influence of a lever in this case, and it is easier wrenched asunder if long.—Accordingly we have found it so; but we have not been able to reduce this influence to calculation.*

constant; that is, $27 : 73.8$, instead of 27 to 61 . It must however be observed, that these ratios obtain only in the particular case for which they are here calculated, in the example where the leverage is constant the general ratio is $(1-n^2)^{\frac{3}{2}} : (1-n^4) :: \text{strength of a solid cylinder} : \text{that of a tube containing the same quantity of matter}$. And it may be very easily proved that the lateral strengths are in the same ratio. (Ed.)

* In wood the lateral adhesion of the fibres being much inferior to their direct cohesion, it is much

[A. 191.] All shafts are exposed to lateral stress and twisting, but it will commonly happen that one of these forces will vastly exceed the other; and consequently, we need only adapt the shaft to the resistance of the greater power; but in the first place we must be able to ascertain when the one or the other must be calculated for.

The resistance of a shaft to a lateral stress must obviously be measured by its stiffness to resist flexure, because it would otherwise play too much on its brasses, couplings, &c. and occasion irregular action in the machine.

From a comparison of shafts in use, it appears, that about the $\frac{1}{100}$ th part of an inch, for each foot in length, is the quantity of flexure that may be allowed without sensibly affecting the regularity of its motion.

In a cast-iron shaft, supposing it to be a

weaker to resist torsion; the fibres sliding one upon another very considerably before the rupture takes place. But this gives the length no sensible influence, when the strain is kept within proper limits. (Ed.)

solid cylinder, if W be the stress when referred to the middle of the length of the shaft; l = the length in feet, and d = the diameter in inches; the deflexion in the middle being $\frac{l}{100}$ of an inch, then $\frac{W l^2}{250} = d^4$. (Essay on Cast Iron, Art. 218.) or $W = \frac{250 d^4}{l^2}$.

[B. 191.] Now if $\frac{740 H}{v}$ be the twisting power collected at the surface of the shaft, where v is the velocity of that surface in feet per second, and H the greatest number of horses' power that is necessary to work the train of machinery to which the shaft belongs, (see note to Art. 78.) we have $124.8 d^2 = \frac{740 H}{6 v}$, (Essay on Cast Iron, Art. 227.) but the velocity of the surface of the shaft is equal to its circumference in feet multiplied by the number of revolutions in a second. And, if this number of revolutions be N , then $v = \frac{11 N d}{42}$; and the equation between the stress and strain, reduces to $d^3 = \frac{3.78 H}{N}$.

It therefore appears, that when $\left(\frac{3.78H}{N}\right)^{\frac{1}{3}} = \left(\frac{Wl^2}{250}\right)^{\frac{1}{3}}$ the strain from torsion will be equal to that from lateral pressure; whence if $\frac{1472}{l^2} \times \left(\frac{H}{N}\right)^{\frac{2}{3}}$ be less than W , the diameter of the shaft must be determined by the rule for lateral stress; if it be greater than W , calculate the diameter by the rule for torsion.

For the advantage of those who are not much versed in calculation, it may be of use to remark, that in common cases when the stress in lbs. multiplied by the square of the length in feet, is less than 3,000; the twisting strain will be the greater; and the reverse.

In a series of lying shafts, as in Fig. 1, Plate V. or Fig. 21, Plate VII. it is necessary to make the journals equal to the twisting strain with the addition of the necessary allowance for wear; hence it will often be necessary to adjust the shafts to both strains: but in all instances where the lateral stress is less than $\frac{1472}{l^2} \times \left(\frac{H}{N}\right)^{\frac{2}{3}}$

the bodies of the shafts need not be greater than the journals.

In other cases, as in Fig. 12, Plate III. the principal twisting strain will be in the part of the shaft between the wheels A and B, while the twisting strain on the gudgeons will be equal only to the friction. (Ed.)

Having thus stated these laws, as far as seems to be necessary for the purpose of our present inquiry, it may be proper next to endeavour to apply them more particularly to practice, with regard to the proportion of shafts.

SECTION IV.

Of the Proportion of Shafts.

192. It was already observed, (Art. 150.) that the gudgeons or journals having to support the whole stress of the shafts to which they belong, their diameters being determined, serve to guide in determining the proportions of the whole shaft. They are subject to wear, which the body seldom

is. They ought, therefore, to be sufficiently large to allow for that wear. It frequently happens, that a shaft has no lateral pressure excepting that which arises from its own weight, for instance in a line of coupled horizontal shafts conveying motion to a distance. In the case of vertical shafts also there is often little or no lateral stress. In such cases, when solid cast-iron shafts are of moderate lengths, it is found from experience, that making them square of the same size throughout,* between the journals, and the measuring a little more on the side than their diameters, gives them sufficient stiffness. Even where there is considerable lateral stress when the shafts are but short, making them in this manner is found to give sufficient stiffness. This square form, in many cases, affords great convenience for hanging or fixing wheels, pulleys, &c. upon them.

193. The gudgeons of water wheels are often so near the wheel, that the stress is,

See note to Art. 136. (Ed.)

in a great measure, taken off the shaft. Hence some water wheels are made without shafts, the gudgeons being fixed to the arms at each side of the wheel. The sole of the wheel, in these cases, may be *considered as a large hollow axle*.*

Cast-iron Shafts.

194. 1st. Let us now suppose a square cast-iron shaft required to be made eight feet long, with gudgeons four inches diameter, having considerable lateral stress in the middle—were the shaft of equal size throughout, it would evidently be weakest

* For a wheel of considerable breadth it will in many instances be an advantage to employ a comparatively small axis, and to render the wheel firm by arms and braces. But if the axis be dispensed with, the same degree of firmness will be gained only by a greater quantity of matter.

The author seems to have had in view the ingenious method of constructing a water wheel executed by Mr. Burns, at Cartside; (Dr. Brewster's edition of Ferguson's Lectures, vol. ii. p. 55.) but the addition of an axis would be of great use if it were only to prevent the racking strain of the gudgeons upon the cross arms. (Ed.)

in the middle. Its section, in that place, ought therefore to be considered. We have already seen (Art. 192) that square shafts, when short, are sufficiently stiff, if made as large as the gudgeons; this shaft may therefore be four inches square at the end. Supposing the section in the middle enlarged to five inches, swelling with a regular curve (Art. 187) from each end (by Prop. II.) then

The cube of 4 is 64

The cube of 5 is 125

that is nearly double.—Therefore, the *strength* of shafts being inversely as their length (Prop. III.) the section as thus increased, would make the eight feet shaft nearly as strong as a shaft subject to the same stress, only four feet long and four inches in the middle.

Let us next see what would be the *stiffness* of this shaft, as thus swelled in the middle to five inches. Then (by Proposition II.) the cube of 4 is 64, which, multiplied by 4 is equal to 256. The cube of 5 is 125, multiplied by 5, is equal to 625;

therefore, the stiffness being inversely as the cube of the lengths—

The cube root of 256 is 6.3

The cube root of 625 is 8.5

Therefore, the section as thus increased, would make the eight feet shaft about as stiff as one of six feet, subject to the same stress, and four inches throughout.

195. 2nd. Let us next suppose the point of greatest lateral pressure to be two feet from one end.

From the properties of the lever, the gudgeon next the point of greatest pressure has three fourths of the whole to sustain.

Now the cube of the diameter of the gudgeon is 64 multiplied by 2, is equal to 128, which represents the strength of both journals taken together.

3-4ths of 128 is equal to 96.

The cube root of 96 is 4.578, the diameter which the largest gudgeon ought to have—

1-4th of 128 is 32.

The cube root of 32 is equal to 3.174,

the diameter which the smallest gudgeon ought to have, or that furthest from the point of pressure.

Having thus obtained the diameters of the two gudgeons, each end of the shaft may be made square, equal to its respective gudgeon (Art. 192).

The stress on the shaft being as the rectangle of the parts, (Prop. V); in this case it is less than the former, as twelve is to sixteen—for four feet multiplied by four feet is equal to 16, which expresses the stress when in the middle—6 feet multiplied by two feet is equal to 12, which expresses the stress when two feet from one end.

This shaft would therefore be as *strong* as the former, if its greatest section were reduced to 4.54:

For the cube of 5 is 125, as $16 : 12 :: 125 : 93.75$, the cube of which is 4.54.

It would be as *stiff* as the former, if its greatest section were reduced to about 4.65:

For the cube of 5 is equal to 125; mul-

multiplied by 5 is equal to 625, which expresses the stiffness of the former shaft, as $16 : 12 :: 625 : 468.75$, which expresses the required stiffness ; and the cube of 4.65, is equal to 100.5444625, which, multiplied by 4.654, is equal to 467.934, which is nearly equal to the required stiffness.

By examining shafts in this way, the millwright, according to the nature of the case, will be enabled to judge how much they should swell at the place of *greatest stress*.

[A. 195.] As far as regards the strength these examinations will apply ; but not to stiffness ; because the rule for the stiffness supposes the shaft to be every where of the same section. And it may be here remarked that a cylinder is stiffer than any figure which can be inscribed within it ; hence there is not that advantage in diminishing a shaft towards the points of support which many people have imagined.

Supposing the reader to have considered

the principles of estimating the pressure on a shaft (Art. [A. 149]) with sufficient attention to be able to ascertain the greatest stress upon one with as much precision as is necessary in these calculations; I will here give rules for computing the diameters of different forms of cast-iron shafts to resist lateral stress.

Cylindrical Shafts of Cast Iron.

If the stress be in the middle, and equal to W cwts. the flexure in the middle not to exceed as many hundredths of an inch as the shaft is feet in length; then by Art. [A. 191] $(\frac{1}{2}W)^{\frac{1}{4}} \times l^{\frac{1}{2}} = d$ the diameter in inches. That is, the fourth root of half the stress in cwts. multiplied by the square root of the length in feet is equal to the diameter in inches.

[B. 195.] If a cylindrical shaft has no other lateral stress to sustain than its own weight, then by making the proper substitutions, the rule becomes $\sqrt{.007 l^3} = d$, the diameter in inches.

That is, multiply the cube of the length in feet by $\cdot 007$; and the square root of this product is the diameter in inches.

The rule now stated may be easily applied to any case with the advantage of both accuracy and simplicity, because it enables us in every instance to include the effect of the weight of the shaft itself; when it is computed in a tabular form. Let the stress, supposed to be at the middle, be n times the weight of the shaft, then $\sqrt{\cdot 012 l^3 n} = d$ in inches.

C. 195. *Table of Shafts of Cast Iron to resist lateral Pressure.*

Length in feet.	Diameter in inches.	Diameter in inches.	Diameter in inches.	Diameter in inches.	Diameter in inches.
2	$\cdot 237$	$\cdot 31$	$\cdot 44$	$\cdot 54$	$\cdot 62$
4	$\cdot 67$	$\cdot 88$	1.24	1.5	1.76
6	1.23	1.61	2.28	2.79	3.22
8	1.9	2.48	3.51	4.30	4.96
10	2.65	3.47	4.9	6.0	6.93
12	3.48	4.55	6.44	7.89	9.10
14	4.38	5.74	8.12	9.94	11.48
16	5.36	7.01	9.92	12.15	14.02
	Own weight only.	Stress equal to its own weight, or $n=1$.	Stress double its own weight, or $n=2$.	Stress three times its own weight, or $n=3$.	Stress four times its own weight, or $n=4$.

Hollow Cylindrical Shafts of Cast Iron.

[D. 195.] To compute the diameter of a shaft when it is to be a hollow cylinder, it is necessary to assume the ratio between the diameter of the hollow part and that of the exterior of the shaft, in order to avoid a complicated rule. Let D be the exterior diameter, and $N D$ the interior one, then W being in cwts. the rule will be $\left(\frac{W l^2}{2(1-N^4)}\right)^{\frac{1}{4}} = D$ the diameter in inches.

But if the shaft supports n times its own weight, then $\sqrt{\frac{\cdot 012 l^3 n}{1+N^2}} = D$, when the necessary substitutions are made. If however some convenient number be fixed upon for N , the rule may be still further simplified. And I think one that will be well adapted for practice is to make the hollow part six-tenths of the exterior diameter. According to this proportion, the rule is $\sqrt{\cdot 009 l^3 n} = D$. That is, the cube of the length in feet, multiplied by $\cdot 009$, and also by the number of times the weight of the

shaft is contained in the stress, then the square root of this product is the diameter in inches.

The weight of the shaft in lbs. will be very nearly equal to the square of the exterior diameter in inches, multiplied by 1.6 times the length in feet.

[E. 195.] *Table of Hollow Shafts of Cast Iron to resist lateral stress.*

Length.	Exterior diameter in inch.	Interior diameter in in.	Exterior diameter in inch.	Interior diameter in in.	Exterior diameter in inch.	Interior diameter in inches.	Exterior diameter in inch.	Interior diameter in inches.
4	1.5	0.9	1.9	1.1	2.2	1.3	2.4	1.4
6	2.8	1.6	3.5	2.1	4.0	2.4	4.5	2.7
8	4.3	2.5	5.3	3.1	6.1	3.6	6.9	4.1
10	6.0	3.6	7.4	4.4	8.5	5.0	9.5	5.7
12	7.9	4.7	9.8	5.8	11.2	6.7	12.6	7.5
14	10.0	6.0	12.3	7.3	14.2	8.5	15.9	9.5
16	12.2	7.3	15.0	9.0	17.3	10.3	19.4	11.6
	Stress four times the weight of the Shaft.		Stress six times the weight of the Shaft.		Stress eight times the weight of the Shaft.		Stress ten times the weight of the Shaft.	

It will be necessary to refer to the tables of resistance to torsion, previous to fixing on the diameter of a shaft.

Wrought-iron Shafts to resist lateral Stress.

[F. 195.] If the diameter of the shaft be calculated for cast iron, and this diameter be multiplied by $\cdot 935$, the product will be the diameter of a wrought-iron shaft of equal stiffness.

For in order that shafts of these metals may be equally stiff, the stiffness of wrought being $1\cdot3$, when that of cast iron is unity, (see Art. 162. note) $c^4 \times 1 = w^4 \times 1\cdot3$. or $c \times \cdot 935 = w$; where c is the diameter of the cast-iron shaft, and w that of the wrought-iron one. (Ed.)

Wooden Shafts.

196. Suppose a cast-iron shaft five inches square at the point of greatest lateral stress—Required the size it should be when made of oak to have the same strength?

The cube of 5 is 125.

Cast-iron is four times stronger than oak, (see Banks on Powers of Machines, &c. p. 94.) therefore $125 \times 4 = 500$.

The cube root of 500 is $7\cdot93$ inches.

[A. 196.] Shafts should be compared by their stiffness, or their resistance to torsion ; but here the question is the comparative stiffness. Now the stiffness of good oak is to that of cast iron as $\frac{1}{11.2} : 1$ (Essay on Cast Iron, Art. Oak.) Therefore $c^4 \times 1 = o^4 \times \frac{1}{11.2}$ when the shafts are of equal stiffness ; hence $1.83 c = o$; where c is the diameter of the cast-iron shaft, and o that of the oak one. And consequently an oak shaft should be 1.83 times the diameter of a cast-iron one to be equally stiff.

And the stiffness of square shafts being as the fourth powers of the sides of the shafts, the side of an oak shaft should be 1.83 times that of an iron one to be of equal stiffness. (Ed.)

197. Required the size of a fir shaft to have the same strength as one of cast iron ?

The cube of 5 is 125.

Cast iron — 5.5 times stronger than fir, (Banks.) therefore $125 \times 5.5 = 687.5$; the cube root of 687.5 is equal to 8.82 ;—*Ans.*

[A. 197.] The stiffness of red or yellow fir is to that of cast iron as $\frac{3}{26} : 1$ (Essay on Cast Iron, Art. Fir.) Therefore c being the diameter of a cast-iron shaft, and f that of a fir one to resist the same stress, $c^4 \times 1 = f^4 \times \frac{3}{26}$, or $c \times \left(\frac{26}{3}\right)^{\frac{1}{4}} = f$, that is $1.716 c = f$. Whence it appears that a shaft of yellow fir should be 1.716 times the diameter of a cast iron one, to resist the same stress.

Shafts that are square should be in the same ratio. (Ed.)

198. In the same manner we might examine various kinds of cylindrical shafts, but I presume what I have already said will suffice. In order to shew, however, that considerable allowance should be made for accidental stress, I may mention the following fact which lately occurred.

199. The hollow shaft of a water wheel, in consequence of one of the gudgeons getting loose, broke quite through near one

end, although it was 16 inches diameter, and two inches thick in the shell—(See Art. 153, 1st table of gudgeons.) The gudgeons of this shaft were not secured by flanches.

200. Hollow cylindrical shafts are often made of equal size throughout, in order to get large flanches, the better to secure the gudgeon.

Shafts subject to Torsion.

201. We saw (Art. 191) that the length of a cast-iron shaft has no influence on its resistance to torsion, and I have already said all that seems to be necessary respecting them. The case is, however, different with regard to wooden shafts ; but we are yet in want of facts to reduce the influence which their length may have, to calculation. As it may give some idea of this influence, I beg leave to state one fact which came recently under my observation.

A shaft about 15 feet long, made of fir, had cast-iron journals $2\frac{1}{2}$ diameter ; one of

the journals broke from fair stress after working about 16 years ; for a considerable time before it broke, the resistance was equal to 7 horses' power, making $11\frac{1}{2}$ revolutions per minute. It gradually wore until it broke ; when that happened, the shaft seemed strained to the utmost, so that it might be reckoned as just equal in strength to the gudgeon ; the shaft was $9\frac{3}{4}$ inches square.

The cube of $9\frac{3}{4}$ is 926,
926 divided by 5.5 (Art 197.) is 16.8 ;

Cube root of 16.8 is 2.6. That is, the fir shaft would be equal to resist the same *lateral stress* as a square cast-iron shaft 2.6 on the side.

The cross-tailed gudgeons of wooden shafts commonly require them to be made sufficiently large to withstand the stress which is brought upon them ; often indeed, formerly, they were much weakened by mortices cut through them for inserting the arms of wheels ; this practice is now, however, almost entirely abandoned.

[A. 201.] The power of a shaft to resist

torsion, has been calculated in comparing the resistance to torsion, with that to lateral stress in Art. [B. 191,] therefore it only remains to apply the equation.

Of Cylindrical Shafts of Cast Iron to resist Torsion.

We have found the equation expressing the relation between the stress and strain to be $\frac{3.78 H}{N} = d^3$, the diameter in inches. But it will be accurate enough for our purpose to increase the constant multiplier to 4, in order to render the computation easier; with this change, when N is the revolutions per minute, we have as a practical rule $\frac{60 \times 4 H}{N} = d^3$, or $\left(\frac{240 H}{N}\right)^{\frac{1}{3}} = d$. That is, divide 240 times the number of horses' power by the number of revolutions per minute, and the cube root of the quotient will be the diameter of the shaft in inches.

But the reader must remember that when in this or any other case, he represents the power of the first mover by a certain number of horses, he must be certain to make

an ample allowance for any temporary increase of the action of the moving power.

The following table is calculated by the above rule, and it will be found useful to compare with the table, Art. [A. 195.]

[B. 201.] *Table of Cylindrical Shafts of Cast Iron to resist Torsion.*

Diameter of Shafts in inches.	Revolutions of the Shafts in a Minute.					
	5 rev.	10 rev.	20 rev.	30 rev.	40 rev.	50 rev.
	Horses' power.	Horses' power.	Horses' power.	Horses' power.	Horses' power.	Horses' power.
2	0.17	0.33	0.66	0.99	1.33	1.66
3	0.56	1.13	2.25	3.37	4.5	5.62
4	1.33	2.66	5.33	7.99	10.66	13.33
5	2.6	5.2	10.4	15.6	20.8	26.0
6	4.5	9.0	18.00	27.0	36.0	45.0
7	7.15	14.3	28.6	42.9	57.2	71.5
8	10.66	21.33	42.66	64.0	85.0	106.6
10	20.83	41.66	83.33	125.0	166.0	208.3
12	36.00	72.00	144.0	216.0	288.0	360.0
14	63.83	127.66	255.33	383.0	510.0	638.3
16	85.33	170.66	341.33	512.0	682.0	853.3

[C. 201.] The same table will serve for hollow cylindrical shafts to resist torsion, if the diameter be multiplied by 1.05, and the diameter of the hollow part be six-tenths of the exterior diameter; for in that case $\left(\frac{1}{1-n^4}\right)^{\frac{1}{3}} = 1.05$. (see Art. 190, note.)

	inches	inches	inches	inches	inches
Shafts of solid cylinders ; diameters	8·	10·	12·	14·	16·
Hollow shafts of } exterior diameter	8·4	10·5	12·6	14·7	16·8
equal strength ; } interior diameter	5·	6·3	7·5	8·8	10·

[D. 201.] This table applies to vertical shafts, but in horizontal ones there is an additional stress if it be only from their own weight, and much more should there be wheels on the shaft. Where the lateral stress is small, it may be allowed for by adding something to the diameter shown by the table, or it may be calculated by the rule at the end of this article.

EXAMPLE.—Suppose a vertical shaft is to make 20 revolutions per minute, the power of the first mover being equal to 18 horses. Look in the column of horses' power under 20 revolutions ; and opposite 18, the diameter will be found in the first column to be 6 inches, for cast iron.

[E. 201.] If the shaft is to be of wrought iron, then multiply the diameter found by the rule or the table, by 0.963 (Art. 162, note.) Thus in the above example, $6 \times 0.963 = 5.778$ inches, the diameter of a wrought-iron shaft to make 20 revolutions

per minute, when the power of the first mover is equal to 18 horses.

[F. 201.] If the shaft be of oak, then the power of oak being $\frac{1}{11.2}$ when that of cast iron is 1,* (Art. A. 196.) and the resistance to torsion being as the cube of the diameter, we have $(11.2)^{\frac{1}{3}} = 2.238$; and multiply the diameter found by the rule or table for cast-iron shafts by 2.238, and it will be the diameter for an oak shaft. Thus in the preceding example, $6 \times 2.238 = 13.428$ inches for the diameter of an oak shaft to make 20 revolutions per minute, the first mover being equal to 18 horses.

[G. 201.] When fir is to be used for a shaft, its diameter should be $\left(\frac{26}{3}\right)^{\frac{1}{3}}$ times that of a cast-iron one for the same purpose; but $\left(\frac{26}{3}\right)^{\frac{1}{3}} = 2.06$ nearly; therefore it should be 2.06 times the diameter of the cast-iron one.

* The relative stiffness is used instead of the relative strength, to reduce the quantity of torsion in wooden shafts.

EXAMPLE.—Let the moving force be equal to 7 horses, and the number of turns per minute $11\frac{1}{2}$, (see the case cited in Art. 201.) then by the rule $\frac{7 \times 240}{11.5} = 146.09$, and the cube root of 146.09 is 5.267 nearly; or practically 5.3 inches should be the diameter of the shaft were it of cast iron. And $2.06 \times 5.3 = 10.918$ inches, or nearly 11 inches for the diameter of a fir shaft. It seems that a shaft of $9\frac{1}{4}$ inches square, of fir, was found equal to the strain; and one 11 inches diameter is at least $\frac{1}{4}$ stronger. I have made these calculations directly from the theory of equal cohesion, but it is so well a known fact that the lateral cohesion of fir is vastly inferior to the direct cohesion, that in the rule for fir shafts to resist torsion, an increase of diameter should be allowed by considering the number of horses' power about $\frac{1}{2}$ more than it is intended to be; at least, till experiment shall have given the precise effect of lateral cohesion in decreasing the force of shafts to resist torsion.

[H. 201.] If a shaft have to sustain both lateral stress and torsion, then the sum of the straining forces must be taken; and hence by Art. [A. 195,] and [A. 201,] we have $\frac{240 N}{N} + \frac{W l^2}{2 d} = d^3$. But in this equation it is difficult to calculate the value of the diameter, as it is what algebraists call an equation of the fourth degree. This difficulty may however be easily avoided by considering $2 d$ to be 2 only, for then the error will always be in excess, except when d is less than unity; and it is much better to be in excess than defect. Consequently we have as a practical rule $\left(\frac{240 H}{N} + \frac{W l^2}{2}\right)^{\frac{1}{3}} = d$, the diameter of the shaft in inches, when of cast iron. Where l is the length in feet between the bearings, H the number of horses which are equal the power of the first mover, N the number of revolutions to be made by the shaft in a minute, and W the lateral stress in cwts.

EXAMPLE.—Suppose that a cylindrical shaft of cast iron is to make 34 revolutions per minute, the power of the first mover

being equal to 3 horses, the length of the shaft 8 feet, and the lateral stress 3 cwts., when reduced to the middle point, then

$$\left(\frac{240 \times 3}{34} + \frac{3 \times 8^2}{2}\right)^{\frac{1}{4}} = (21.18 + 96)^{\frac{1}{4}} = (117.18)^{\frac{1}{4}} = 4.893 \text{ inches. (Ed.)}$$

202. With regard to the making of patterns of cast-iron shafts, I beg leave to refer the reader to what I have said in Essay I. page 219 to 229, relative to the making of patterns for cast-iron wheels, which is, in a great measure, applicable to those of shafts. Nor do I recollect any thing on this subject to add here; only I would remind the millwright to make the allowance for contraction of metal of one-eighth of an inch to the foot in the pattern.

203. The following table contains the dimensions of shafts subject to torsion, and to considerable lateral pressure, as they were executed by a respectable millwright. It will serve to shew the sizes of the parts as found in practice sufficiently strong,

and may be found useful to compare with those which would be produced, calculating on the principles laid down in this Essay.

Column 6th, therefore, shews the diameters which these journals ought to have, were 400 used as the multiplier.—(See Art. 171.)

Table of Shafts.

	1	2	3	4	5	6	
Name.	Horses' power.	Revolutions per minute.	Diameter of journals in inches.	Length of Shaft.	Section in the middle.	Diameters of journals by calculating from a multiplier of 400.	Remarks.
Cast-Iron Lying Shaft.	20	20	6	11'	7 $\frac{3}{4}$	7,368	Feathered Shafts.
	18	22	5 $\frac{3}{4}$	11'	7	6,889	
	16	22	5 $\frac{1}{4}$	10'6	7	6,621	
	14	24	5	10'	6 $\frac{3}{4}$	6,153	
	12	25	5	9'6	6	5,768	
	10	25	4 $\frac{3}{4}$	9'	6 $\frac{1}{4}$	5,428	
	8	27	4	9'	6	4,904	
	6	28	4 $\frac{1}{4}$	8'6	5 $\frac{3}{4}$	4,414	
	5	30	3 $\frac{7}{8}$	8'6	5 $\frac{1}{4}$	4,061	
	4	32	3 $\frac{3}{4}$	8'	4 $\frac{3}{4}$	3,484	
	3	34	3 $\frac{1}{4}$	8'	4	1,203	
	2	46	2 $\frac{3}{4}$	8'	3	2,802	
	1	40	2	8'	3 $\frac{1}{4}$	2,154	
Malleable-Iron Lying Shaft.	6	28	3	8'6			Square Shafts.
	5	30	3 $\frac{1}{4}$	8'			
	4	32	2	8'			
	3	34	2	8'			
	2	36	1 $\frac{3}{4}$	8'			
	1	40	1	7'6			

APPENDIX.

COHESIVE STRENGTH OF DIFFERENT METALS.

204. “ We shall take for the measure of cohesion the number of pounds avoirdupois which are just sufficient to tear asunder a rod or bundle of one inch square. From this it will be easy to compute the strength corresponding to any other dimension.

“ 1st, Metals.

	lbs.
“ Gold cast	{ 20,000 24,000
Silver cast	{ 40,000 43,000
Copper cast {	Japan 19,500
	Barbary 22,000
	Hungary 31,000
	Anglesea 34,000
	Sweden 37,000
Iron cast	{ 42,000 59,000

		lbs.
Iron bar	{ Ordinary	68,000
	{ Stirian	75,000
	{ Best Swedish and Russian	84,000
	{ Horse nails	71,000
Steel bar	{ Soft	120,000
	{ Razor temper	150,000
Tin cast	{ Malacca	3,100
	{ Banca	3,600
	{ Block	3,800
	{ English block	5,200
	{ ——— grain	6,500
Lead cast		860
Regulus of Antimony		1,000
Zinc		2,600
Bismuth		2,900

“The only author who has put it in our power to judge of the propriety of his experiments is Muschenbroëk. He has described his method of trial minutely, and it seems unexceptionable. The woods were all formed into slips fit for his apparatus, and part of the slip was cut away to a parallelopiped of $\frac{1}{8}$ of an inch square, and therefore $\frac{1}{75}$ of a square inch in section. The absolute strengths of a square inch were as follow.

	lbs.		lbs.
Locus tree	20,100	Pomegranate	9,750
Jujeb	18,500	Lemon	9,250
Beech Oak	17,300	Tamarind	8,750
Orange	15,500	Fir	8,330
Alder	13,900	Walnut	8,130
Elm	13,200	Pitch Pine	7,640
Mulberry	12,500	Quince	6,750
Willow	12,500	Cypress	6,000
Ash	12,000	Poplar	5,500
Plum	11,800	Cedar	4,880
Elder	10,000		

“Mr. Muschenbroëk has given a very minute detail of experiments on the ash and the walnut, stating the weights which were required to tear asunder slips taken from the four sides of the tree, and on each side in a regular progression from the centre to the circumference. The numbers of this table corresponding to these two timbers may therefore be considered as the average of more than 50 trials made of each, and he says that all the others were made with the same care. We cannot therefore see any reason for not confiding in the results; yet they are consider-

ably higher than those given by some other other writers. Mr. Pitot says, on the authority of his own experiments, and of those of Mr. Parent, that 60 pounds will just tear asunder a square line of sound oak, and that it will bear 50 with safety. This gives 8640 for the utmost strength of a square inch, which is much inferior to Muschenbroëk's valuation.

“ We may add to these

	cwt. †
Ivory	16,280
Bone	5,250
Horn	8,750
Whalebone	7,500
Tooth of sea calf	4,075

“ The reader will surely observe that these numbers express something more than the utmost cohesion, for the weights are such as will very quickly, that is, in a minute or two, tear the rods asunder. It may be said in general that two thirds of these weights will sensibly impair the strength after a considerable while; and that one half is the utmost that can remain

suspended at them, without risk, for ever; and it is this last allotment that the engineer should reckon upon in his constructions. There is however a considerable difference in this respect. Woods of a very straight fibre, such as fir, will be less impaired by any load which is not sufficient to break them immediately."

205. "According to Mr. Emerson, the load which may be safely suspended to an inch square is as follows :

	lbs.
Iron	76,400
Brass	35,600
Hempen Rope	19,600
Ivory	15,700
Oak, box, yew, plum-tree	7,850
Elm, ash, beech	6,070
Walnut, plum	5,360
Red fir, holly, elder, plane, crab	5,000
Cherry, hazle	4,760
Alder, asp, birch, willows	4,290
Lead	430
Freestone	91

"He gives us a practical rule, that a cy-

linder whose diameter is d inches, loaded to one fourth of its absolute strength, will carry as follows :

	cwt.
Iron	135
Good rope	22
Oak	14
Fir	9

“The rank which the different woods hold in this list of Mr. Emerson’s is very different from what we find in Muschenbroëk’s. But precise measures must not be expected in this matter. It is wonderful that in a matter of such unquestionable importance the public has not enabled some persons of judgment to make proper trials. They are beyond the abilities of private persons.*

206. Mr. Banks (Powers of Machines, &c. p. 94,) takes iron at an average to be four times as strong as oak, and $5\frac{1}{2}$ times as strong as deal or fir.

207. “According to the experiments of

* En. Brit. Art. Strength of Materials.

various authors, the cohesive strength of a square inch of razor steel is about 150 thousand pounds, of soft steel 120, of wrought iron 80, of cast iron 50, of good rope 20, of oak, beech, and willow wood, in the direction of their fibres 12, of fir 8, and of lead about three thousand pounds; the cohesive strength of a square inch of brick 300, and of free stone 200; teak wood, the *tectona grandis*, is said to be still stronger than oak.

“The strength of different materials in resisting compression, is liable to great variation. In steel and in willow wood, the cohesive and repulsive strength appear to be nearly equal. Oak will suspend much more than fir, but fir will support twice as much as oak, probably on account of the curvature of the fibres of oak. Freestone has been found to support about 2000 pounds for each square inch; oak, in some practical cases, more than 4000.

“The strongest wood of each tree is neither at the centre nor at the circumference, but in the middle between both;

and in Europe it is generally thicker and firmer on the south-east side of the tree. Although iron is much stronger than wood, yet it is more liable to accidental imperfections ; and when it fails, it gives no warning of its approaching fracture. The equable equality of steel may be ascertained by corrosion in an acid, but there is no easy mode of detecting internal flaws in a bar of iron, and we can only rely on the honesty of the workmen for its soundness. Wood, when it is crippled, complains, or emits a sound, and after this, although it is much weakened, it may still retain strength enough to be of service.*

[A. 207.] The cohesive force of metals has been examined by several experimental inquirers, besides those noticed in the extracts made by our author, and our knowledge of this subject has been recently extended very considerably by the experiments of Telford, Brown, and Rennie. I

* Young's Nat. Phil. vol. i. p. 151.

Table of Experiments on the Direct Cohesion of Metals.

Description of metal.	Force (in lbs.) that would tear a- sunder a bar of 1 in. sq.	Experimentalist.	Quoted from.
I. STEEL.			
Cast steel, previously tilted	134,256	Rennie	Phil. Mag. vol. liii. p. 167.
Cast steel	63,065	Brown	Barlow's Essay, p. 234.
Blister steel, reduced by the hammer ..	133,152	Rennie	Phil. Mag. vol. liii. p. 167.
Blister steel	32,973	Brown	Barlow's Essay, p. 234.
Shear steel, reduced by the hammer ..	127,632	Rennie	Phil. Mag. vol. liii. p. 167.
II. MALLEABLE IRON.			
Iron wire	113,077	Sickingen	Ann. de Chimie, vol. xxv. p. 9.
Iron wire	93,964	Telford	Barlow's Essay, p. 222.
Iron wire	85,797	Buffon	Œuvres de Gauthey, ii. 153.
German bar, mark B R, highest result }	93,069	Muschenbroëk	Intro. ad Phil. Nat. i. 426.
Swedish bar, highest result	88,972	idem	
German bar, mark L, highest result . }	85,900	idem	
Liege bar, highest re- sult	82,839	idem	
Spanish bar	81,901	idem	
Bar	80,833	Soufflot	Rondelet's L'Art. de Bâtir, iv. 500.
Oosement bar, high- est result	76,697	Muschenbroëk	Intro. ad Phil. Nat. i. 426.
Swedish bar, reduced by the hammer ..	72,064	Rennie	Phil. Mag. vol. liii. p. 167.
Common round iron ..	71,300	Telford	Barlow's Essay, p. 230.
German bar, mark L .	69,538	Muschenbroëk	Intro. ad Phil. Nat. i. 426.
Common Stafford- shire bar	69,440	Telford	Barlow's Essay, p. 230.
Common German bar .	69,133	Muschenbroëk	Intro. ad Phil. Nat. i. 426.
Swedish bar	68,728	idem	
Oosement bar, the same			
Welsh bar	66,752	Telford	Barlow's Essay, p. 228.
Bar of the best quality	66,000	Rumford	Phil. Mag. x. 51.
A bar of Welsh, one of Swedish,* and one faggoted scrap Iron, each gave a result of	64,960	Telford	Barlow's Essay, p. 229.

* The Swedish bar broke at a flaw.

Table continued.

Description of metal.	Force (in lbs.) that would tear a bar of 1 in. sq.	Experimentalist.	Quoted from.
Liege bar	62,369	Muschenbroëk	Intro. ad Phil. Nat. i. 426.
Staffordshire bar	61,600	Telford	Barlow's Essay, p. 229.
German bar, mark } BR	61,361	Muschenbroëk	Intro. ad Phil. Nat. i. 426.
Bar (mean of 33 expts.)	61,041	Perronet	Ceuvres de Gauthey, ii. 154.
Russian old sable, } mark CCN	59,472	Brown	Barlow's Essay, p. 233.
English bar, reduced } by the hammer.	55,872	Rennie	Phil. Mag. vol. liii. p. 167.
Welsh bar, (3 expts.) ..	55,776	Brown	Barlow's Essay, p. 233.
Bar of good quality ..	55,000	Rumford	Phil. Mag. vol. x. p. 51.
Swedish bar, (3 expts.)	53,244	Brown	Barlow's Essay, p. 232.
Bar (fine grain)	49,982	Rondelet	L' Art de Bâtir, iv. 502.
— (medium fineness)	34,081	idem	
— (coarse grained)	20,460	idem	
III. CAST IRON.			
Bar, spec. grav. 7·807	68,295	Muschenbroëk	Intro. ad Phil. Nat. i. 417.
Bar, cast vertically ..	19,488	Rennie	Phil. Mag. vol. liii. p. 167.
Bar, cast horizontally	18,656	Rennie	idem
Bar, Welsh pig	16,264	Brown	Barlow's Essay, p. 235.
IV. COPPER.			
Wire	61,228	Sickingen	Ann. de Chimie xxv. 9.
Wrought copper, re- duced by the ham- mer	33,792	Rennie	Phil. Mag. vol. liii. p. 167.
Cast, Barbary, spec. grav. 8·182	22,570	Muschenbroëk	Intro. ad Phil. Nat. i. 417.
Cast, Japan, spec. grav. 8·726	20,272	idem	
Cast	19,072	Rennie	Phil. Mag. vol. liii. p. 167.
V. PLATINUM.			
Platinum wire, spec. grav. 20·847	56,473	Morveau	Ann. de Chimie, xxv. 8.
Platinum wire	52,987	Sickingen	idem, p. 9.
VI. SILVER.			
Silver wire	38,257	Sickingen	Ann. de Chimie, vol. xxv. 9.
— cast, spec. grav. 11·091	40,902	Muschenbroëk	Intro. ad Phil. Nat. i. 417.

Table continued.

Description of metal.	Force (in lbs.) that would tear a- sunder a bar of 1 in. sq.	Experimentalist.	Quoted from.
VII. GOLD.			
Gold wire	30,888	Sickingen	Ann. de Chimie, vol. xxv. 9.
— cast, spec. grav. } 19·238	20,450	Muschenbroëk	Intro. ad Phil. Nat. i. 417.
VIII. ZINC.			
Zinc wire	22,551	Morveau	Ann. de Chimie, v. lxxi. p. 194.
— sheet	16,600	Tredgold	Phil. Mag. vol. l. p. 422.
— cast	2,689	Muschenbroëk	Intro. ad Phil. Nat. i. 407.
IX. TIN.			
Tin wire	7,129	Morveau	Ann. de Chimie, v. lxxi. p. 194.
English block, cast ..	6,650	Muschenbroëk	Intro. ad Phil. Nat. i. 417.
— spec. grav. 7·295	5,322	idem	
Cast	4,736	Rennie	Phil. Mag. vol. liii. p. 167.
Banca tin, cast, spec. } grav. 7·2165	3,679	Muschenbroëk	Intro. ad Phil. Nat. i. 417.
Malacca tin, cast, } spec. grav. 6·1256 }	3,211	idem	
X. LEAD.			
Milled sheet, spec. } grav. 11·407	3,328	Tredgold	Phil. Mag. vol. l. p. 422.
Wire	3,146	Muschenbroëk	Intro. ad Phil. Nat. i. 452.
Wire, spec. grav. } 11·282	2,581	idem	
Wire	2,547	Morveau	Ann. de Chimie, v. lxxi. p. 194.
Cast lead	1,824	Rennie	Phil. Mag. vol. liii. p. 167.
— English, spec. } grav. 11·479	885	Muschenbroëk	Intro. ad Phil. Nat. i. 452.
XI. BISMUTH.			
Bismuth, cast, spec. } grav. 9·810	3,250	Muschenbroëk	Intro. ad Phil. Nat. i. 417.
— spec. grav. } 9·926	3,008	idem	
XII. ANTIMONY.			
Antimony, cast, spec. } grav. 4·500	1,060	Muschenbroëk	Intro. ad Phil. Nat. i. 417.

As this table, the most extensive of the kind, exhibits at one view the chief results of the experiments on the direct cohesion of the metals in English avoirdupois pounds when the area is a superficial inch, as well as references to the works wherein those experiments are described, it will be useful to direct the labours of future inquirers to such experiments as are best adapted to increase or correct our knowledge on this subject. It was collected at various times for my own information, and I hope it will be equally useful to others. When experiments are not reduced to a common standard, they cannot be compared without much labour: in the original descriptions of these experiments, this has not been done; they are described chiefly as they were made, and for further information, to these descriptions I must refer the reader. If he be interested in these researches, the works of Muschenbroëk, Rondelet, and Barlow, and Rennie's paper in the *Philosophical Transactions*,* will afford him much information.

* The results are quoted in the table from the Philo-

[B. 207.] From the simple metals we naturally look to the alloys, some of which are of much importance. Here the curious but important fact that the union of two metals produces a compound of greater tenacity than either of the metals it is formed of, will be noticed.

Table of Experiments on the direct Cohesion of Alloys.

Alloy of				Force (in lbs.) that would tear asunder a bar of one inch square.	Specific gravity of the alloy.	Experimentalist.
Parts.		Parts.				
Copper	10	Tin	1	32,093	8.351	Muschenbroëk Intro. ad Phil. Nat.
_____	8	—	1	36,088	8.392	idem
_____	6	—	1	44,071	8.707	idem
_____	4	—	1	35,739	8.723	idem
_____	2	—	1	1,017		idem
Gun metal, hard				36,368		Rennie Phil. Trans.
Brass, fine yellow				17,968		idem
Tin, English	10	Lead	1	6,904		Muschenbroëk
_____	8	_____	1	7,922		idem
_____	6	_____	1	7,997		idem
_____	4	_____	1	10,607		idem
_____	2	_____	1	7,470		idem
_____	1	_____	1	7,074		idem
Tin, Banca	10	Antimony	1	11,181	7.359	Muschenbroëk
_____	8	_____	1	9,881	7.276	idem

sophical Magazine, because it is a less expensive work than the Transactions.

Table continued.

Alloy of		Force (in lbs.) that would tear a under a bar of one inch square.	Specific gravity of the alloy.	Experimentalist.
Parts.	Parts.			
Tin, Banca 6	Antimony 1	12,632	7.228	Muschenbroëk
_____ 4	_____ 1	13,480	7.192	idem
_____ 2	_____ 1	12,029	7.105	idem
_____ 1	_____ 1	3,184	7.060	idem
Tin, Banca 10	Bismuth 1	12,688	7.576	Muschenbroëk
_____ 4	_____ 1	16,692	7.613	idem
_____ 2	_____ 1	14,017	8.076	idem
_____ 1	_____ 1	12,020	8.146	idem
_____ 1	_____ 2	10,013	8.580	idem
_____ 1	_____ 4	7,875	9.009	idem
Tin, Banca 10	Zinc, Indian 1	12,914	7.288	Muschenbroëk
_____ 2	_____ 1	15,025	7.000	idem
_____ 1	_____ 1	15,844	7.321	idem
_____ 1	_____ 2	16,023	7.100	idem
_____ 1	_____ 10	5,671	7.130	idem
Tin, English 8	Zinc, Goslar 1	10,607		Muschenbroëk
_____ 4	_____ 1	10,258		idem
_____ 2	_____ 1	10,964		idem
_____ 1	_____ 1	9,024		idem
Tin, English 1	Antimony 1	1,450	7.000	Muschenbroëk
_____ 3	_____ 2	3,184		idem
_____ 4	_____ 1	11,343		idem
Lead, Scotch 1	Bismuth 1	7,319	10.931	Muschenbroëk
_____ 2	_____ 1	5,840	11.090	idem
_____ 10	_____ 1	2,826	10.827	idem

Brass is an alloy of copper and zinc, gun metal is an alloy of copper and tin, sometimes in the proportion of 96 parts of copper to 11 parts of tin, but perhaps more usually 108 parts of copper to 11 parts of

tin.* It will be seen by this table that the proportion of six of copper to one of tin, is the most tenacious compound. A proportion very near to this is used for bearings, bushes, and some purposes in machinery; but it is too hard and brittle for many uses. It is worthy of remark, that copper and tin are soft and malleable metals, but when combined, they form a tenacious, brittle, and hard alloy. Both the hardness and brittleness is increased by augmenting the proportion of tin.

Tables of the cohesive force of woods of various kinds may be seen in my Elementary Principles of Carpentry, Sect. II. also in Muschenbroëk's work above quoted, or in Barlow's Essay on the Strength of Timber. (Ed.)

* These numbers give the nearest chemical proportions to those in use among founders. For further information on this subject, see the art. Brass, Supplement to Ency. Brit.

tin. It will be seen by this table that the proportion of six of copper to one of tin, is the most tenacious compound. A proportion very near to this is used for bearings, bushes, and some purposes in machinery; but it is too hard and brittle for many uses. It is worthy of remark, that copper and tin are soft and malleable metals, but when combined, they form a tenacious, brittle, and hard alloy. Both the hardness and brittleness is increased by augmenting the proportion of tin.

Tables of the cohesive force of woods of various kinds may be seen in my Elementary Principles of Carpentry, Sect. II. also in Mueschbrock's work above quoted, as in my Essay on the Strength of Timber. (Ed.)

* These numbers give the nearest chemical proportions to those in use among foundries. For further information on this subject, see the art, Brass, Supplement to Encyc. Brit.

Buchanan, Robertson,

Practical essays on mill work and other machinery

Bd.: 1

London 1823

Math.a. 50 m-1

urn:nbn:de:bvb:12-bsb10080599-0